Abstract Coordinate Transforms in Kinematic Changeable Sets and their Properties

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One of the fundamental postulates of the special relativity theory is existence of a single system of universal coordinate transforms for inertial reference frames, that is coordinate transforms, which are uniquely determined by space-time coordinates of a material point. In this paper the abstract mathematical theory of coordinate transforms in kinematic changeable sets is developed. In particular it is proved the formal possibility of existence of kinematics, which do not allow universal coordinate transforms. Such kinematics may be applied for simulation the evolution of physical systems under the condition of hypothesis on existence of particle-dependent velocity of light.

Contents

1 Introduction.

Due to the OPERA experiments conducted within 2011-2012 years [1], quite a lot physical works appeared, in which authors are trying to modify the special relativity theory to agree its conclusions with the hypothesis of existence of objects moving at velocity, greater than the velocity of light. Despite the fact that the superluminal results of OPERA experiments (2011-2012) were not confirmed later, the problem of constructing a theory of super-light movement remains actual within more than 50 last years [2,3]. At the present time existence of a few kinematic theories of tachyon motion generates the problem of construction new mathematical structures, which would allow to simulate of evolution of physical systems in a framework of different laws of kinematics. Under the lack of experimental verification of conclusions for tachyon kinematics theories, such mathematical structures may at least guarantee the correctness of receiving these conclusions in accordance with the postulates of these theories. This paper is devoted to building of these mathematical structures. Investigations in this direction may be also interesting for astrophysics, because there exists the hypothesis, that in large scale of the Universe, physical laws (in particular, the laws of kinematics) may be different from the laws, acting in the neighborhood of our solar System.

On a physical level, the problem about investigation of kinematics with arbitrary space-time coordinate transforms for inertial reference frames, was presented in the [4] for the case, when the space of geometric variables is three-dimensional and Euclidean. The particular case of coordinate transforms, considered in [4] are the (three-dimensional) classical Lorentz transforms as well as generalized Lorentz transforms in the sense of E. Recami [5–7] (for reference frames moving at a velocity greater than the velocity light). In the papers [8, 9] the general definition of linear coordinate transforms and generalized Lorentz transforms is given for the case, where the space of geometric variables is any real Hilbert space.

It should be noted, that mathematical apparatus of the papers [4–9] is not based on the theory of changeable sets, which greatly reduces its generality. In particular, mathematical apparatus of these papers allows only studying of universal coordinate transforms (that is coordinate transforms, which are uniquely determined by the geometrically-time position of the considered object). The present paper is based on the general theory of changeable sets [10–13]. In the paper the definitions of the actual and universal coordinate transform in kinematic changeable sets are presented. We prove, that in classical Galilean and Lorentz-Poincare kinematics the universal coordinate transform always exists. Also in the paper we construct the class of kinematics, in which every particle can have its own "velocity of light" and prove, that, in these kinematics, universal coordinate transform does not exist.

2 Basic Concepts of the Theory of Changeable Sets.

In the present section we recall necessary denotations and concepts of the changeable sets theory, introduced in [11] (see also [10, 12–14, 16, 17]). Changeable and base changeable sets will be denoted by large calligraphy or Gothic letters. Further for any base changeable set $\mathcal{B}$ we use the following denotations:

$\mathfrak{Bs}(\mathcal{B})$ – the basic set or the set of all elementary states of $\mathcal{B}$.
$\mathbb{B}\mathfrak{s}(\mathcal{B})$ – the set of all elementary-time states of $\mathcal{B}$.
$\mathbf{Tm}(\mathcal{B})$ – the set of time points of $\mathcal{B}$.
$\mathbb{T}\mathbf{m}(\mathcal{B}) = (\mathbf{Tm}(\mathcal{B}), \leq_{\mathcal{B}})$, where $\leq_{\mathcal{B}}$ – relation of time order of $\mathcal{B}$.
$\underset{\mathcal{B}}{\leftarrow}$ – directing relation of changes of $\mathcal{B}$.
$\overset{\mathbb{B}\mathfrak{s}}{\underset{\mathcal{B}}{\leftarrow}}$ – base of elementary processes of $\mathcal{B}$.

Elements of the sets $\mathfrak{Bs}(\mathcal{B})$ and $\mathbb{B}\mathfrak{s}(\mathcal{B})$ will be named by elementary or elementary-time states of $\mathcal{B}$ (correspondingly).

Properties 1 ([17]). *Let $\mathcal{B}$ be any base changeable set. Then:*

1. $\underset{\mathcal{B}}{\leftarrow}$ *is reflexive binary relation, defined on $\mathfrak{Bs}(\mathcal{B})$, that is for any elementary state $x \in \mathfrak{Bs}(\mathcal{B})$ the correlation $x \underset{\mathcal{B}}{\leftarrow} x$ is performed.*
2. $\leq_{\mathcal{B}}$ *is relation of (not-strict) linear order defined on $\mathbf{Tm}(\mathcal{B})$ (i.e. $\mathbb{T}\mathbf{m}(\mathcal{B}) = (\mathbf{Tm}(\mathcal{B}), \leq_{\mathcal{B}})$ is linearly (totally) ordered set in the sense of* [15, p. 12]*).*
3. $\mathbb{B}\mathfrak{s}(\mathcal{B}) \subseteq \mathbf{Tm}(\mathcal{B}) \times \mathfrak{Bs}(\mathcal{B})$ *(where $\mathbf{T} \times \mathcal{X} = \{(t,x) \mid t \in \mathbf{T},\, x \in \mathcal{X}\}$ is the Cartesian product of the sets $\mathbf{T}$ and $\mathcal{X}$).*
4. $\overset{\mathbb{B}\mathfrak{s}}{\underset{\mathcal{B}}{\leftarrow}}$ *is reflexive binary relation, defined on $\mathbb{B}\mathfrak{s}(\mathcal{B})$.*

Remark 1. In the case, when the base changeable set $\mathcal{B}$ is known in advance, in the denotations $\leq_{\mathcal{B}}$, $\underset{\mathcal{B}}{\leftarrow}$, $\overset{\mathbb{B}\mathfrak{s}}{\underset{\mathcal{B}}{\leftarrow}}$ the char "$\mathcal{B}$" will be released, and we will use denotations $\leq$, $\leftarrow$, $\overset{\mathbb{B}\mathfrak{s}}{\leftarrow}$ instead. In the cases, when we can uniquely determine from the previous text, that the relation $\overset{\mathbb{B}\mathfrak{s}}{\underset{\mathcal{B}}{\leftarrow}}$ is applied for elementary-time states $\omega_1, \omega_2 \in \mathbb{B}\mathfrak{s}(\mathcal{B})$, we use the denotation $\omega_2 \underset{\mathcal{B}}{\leftarrow} \omega_1$ or, simply, $\omega_2 \leftarrow \omega_1$ instead of $\omega_2 \overset{\mathbb{B}\mathfrak{s}}{\underset{\mathcal{B}}{\leftarrow}} \omega_1$.

Let, $\mathbb{T} = (\mathbf{T}, \leq)$ be any linearly ordered set and $\mathcal{X}$ be any set. For any ordered pair $\omega = (t, x) \in \mathbf{T} \times \mathcal{X}$ we use the following denotations:

$$\mathsf{bs}\,(\omega) := x, \quad \mathsf{tm}\,(\omega) := t. \tag{1}$$

Properties 2 ([11,17]). *Let $\mathcal{B}$,$\mathcal{B}_1$,$\mathcal{B}_2$ be any base changeable sets. Then:*

1. *If $\omega_1, \omega_2 \in \mathbb{B}\mathfrak{s}(\mathcal{B})$ and $\omega_2 \leftarrow \omega_1$, then $\mathsf{bs}\,(\omega_2) \leftarrow \mathsf{bs}\,(\omega_1)$ and $\mathsf{tm}\,(\omega_1) \leq \mathsf{tm}\,(\omega_2)$. If, in addition, we have, $\omega_1 \neq \omega_2$, then $\mathsf{tm}\,(\omega_1) < \mathsf{tm}\,(\omega_2)$.*
2. $\mathfrak{Bs}(\mathcal{B}) = \{\mathsf{bs}\,(\omega) \mid \omega \in \mathbb{B}\mathfrak{s}(\mathcal{B})\}$*;*
3. *For arbitrary $x_1, x_2 \in \mathfrak{Bs}(\mathcal{B})$ the condition $x_2 \leftarrow x_1$ holds if and only if there exist elementary-time states $\omega_1, \omega_2 \in \mathbb{B}\mathfrak{s}(\mathcal{B})$ such, that $\mathsf{bs}\,(\omega_1) = x_1$, $\mathsf{bs}\,(\omega_2) = x_2$ and $\omega_2 \leftarrow \omega_1$.*
4. *If $\mathbb{T}\mathbf{m}\,(\mathcal{B}_1) = \mathbb{T}\mathbf{m}\,(\mathcal{B}_2)$, $\mathbb{B}\mathfrak{s}\,(\mathcal{B}_1) = \mathbb{B}\mathfrak{s}\,(\mathcal{B}_2)$, $\overset{\mathbb{B}\mathfrak{s}}{\underset{\mathcal{B}_1}{\leftarrow}} = \overset{\mathbb{B}\mathfrak{s}}{\underset{\mathcal{B}_2}{\leftarrow}}$, then $\mathcal{B}_1 = \mathcal{B}_2$.*

Remark 2. In some earlier works [8, 11] it was used the synonymic term "basic changeable set" instead of the term "base changeable set".

Further for any changeable set $\mathcal{Z}$ we use the following denotations:

$\mathcal{I}nd\,(\mathcal{Z})$ − the ***index*** set of the changeable set $\mathcal{Z}$.

$\mathcal{L}k\,(\mathcal{Z})$ − the set of all ***areas of perception*** of $\mathcal{Z}$.

Herewith for any areas of perception $\mathfrak{l}, \mathfrak{m} \in \mathcal{L}k\,(\mathcal{Z})$ we will use denotations:

$\mathtt{ind}\,(\mathfrak{l})$ − the index of the area of perception $\mathfrak{l}$.

$\mathfrak{Bs}(\mathfrak{l})$, $\mathbb{B}\mathfrak{s}(\mathfrak{l})$, $\mathbf{Tm}(\mathfrak{l})$, $\leq_{\mathfrak{l}}$, $\underset{\mathfrak{l}}{\leftarrow}$, $\overset{\mathbb{B}\mathfrak{s}}{\underset{\mathfrak{l}}{\leftarrow}}$ — the set of all elementary states of $\mathfrak{l}$, the set of all elementary-time states of $\mathfrak{l}$, the set of time points of $\mathfrak{l}$, relation of time order of $\mathfrak{l}$, directing relation of changes of $\mathfrak{l}$, base of elementary processes of $\mathfrak{l}$ (correspondingly).

$\mathbb{T}\mathbf{m}(\mathfrak{l}) = (\mathbf{Tm}(\mathfrak{l}), \leq_{\mathfrak{l}})$.

$\langle \mathfrak{m} \leftarrow \mathfrak{l}, \mathcal{Z} \rangle$ − ***unification mapping*** from the area of perception $\mathfrak{l}$ into the area of perception $\mathfrak{m}$.

Remark 3. From the definition of changeable set ([13, Definition 3.1], [11, Definition 9.6]) it directly follows, that for any area of perception $\mathfrak{l} \in \mathcal{L}k(\mathcal{Z})$ of any changeable set $\mathcal{Z}$, Properties 1 and Properties 2 are holding, where *we use all abbreviated variants of notations, described in the Remark 1* (but, with replacement of the symbol "$\mathcal{B}$" by the symbol "$\mathfrak{l}$" and the term "base changeable set" by the term "area of perception").

Properties 3 ([17]). *Let $\mathcal{Z}$ be any changeable set. On that:*

1. *The sets $\mathcal{L}k(\mathcal{Z})$ and $\mathcal{I}nd(\mathcal{Z})$ always are nonempty, moreover $\mathcal{I}nd(\mathcal{Z}) = \{\mathtt{ind}(\mathfrak{l}) \mid \mathfrak{l} \in \mathcal{L}k(\mathcal{Z})\}$.*
2. *For arbitrary $\mathfrak{l}, \mathfrak{m} \in \mathcal{L}k(\mathcal{Z})$ the equality $\mathfrak{l} = \mathfrak{m}$ holds if and only if $\mathtt{ind}(\mathfrak{l}) = \mathtt{ind}(\mathfrak{m})$.*
3. *For arbitrary $\mathfrak{l}, \mathfrak{m} \in \mathcal{L}k(\mathcal{Z})$ the unification mapping $\langle \mathfrak{m} \leftarrow \mathfrak{l}, \mathcal{Z}\rangle$ is the mapping from $2^{\mathbb{B}\mathfrak{s}(\mathfrak{l})}$ into $2^{\mathbb{B}\mathfrak{s}(\mathfrak{m})}$, where $2^{\mathbf{M}}$ means the set of all subset of the set $\mathbf{M}$.*

Assertion 1 ([17]). *Let, $\mathcal{Z}_1$, $\mathcal{Z}_2$ be arbitrary changeable sets, moreover, $\mathcal{L}k(\mathcal{Z}_1) = \mathcal{L}k(\mathcal{Z}_2)$ and for any areas of perception $\mathfrak{l}, \mathfrak{m} \in \mathcal{L}k(\mathcal{Z}_1) = \mathcal{L}k(\mathcal{Z}_2)$ it is true the equality: $\langle \mathfrak{m} \leftarrow \mathfrak{l}, \mathcal{Z}_1\rangle = \langle \mathfrak{m} \leftarrow \mathfrak{l}, \mathcal{Z}_2\rangle$. Then, $\mathcal{Z}_1 = \mathcal{Z}_2$.*

Let $\mathfrak{l}, \mathfrak{m} \in \mathcal{L}k(\mathcal{Z})$ be arbitrary areas of perception of the changeable set $\mathcal{Z}$. Then for any subset $A \subseteq \mathbb{B}\mathfrak{s}(\mathfrak{l})$ we denote by $\langle \mathfrak{m} \leftarrow \mathfrak{l}, \mathcal{Z}\rangle A$ the result of action of unification mapping $\langle \mathfrak{m} \leftarrow \mathfrak{l}, \mathcal{Z}\rangle$ on the set A. Hence, $\langle \mathfrak{m} \leftarrow \mathfrak{l}, \mathcal{Z}\rangle A := \langle \mathfrak{m} \leftarrow \mathfrak{l}, \mathcal{Z}\rangle (A)$. In the case, when the changeable set $\mathcal{Z}$ is known in advance, we will release the symbol "$\mathcal{Z}$" in the denotation $\langle \mathfrak{m} \leftarrow \mathfrak{l}, \mathcal{Z}\rangle$, using instead of it the denotation $\langle \mathfrak{m} \leftarrow \mathfrak{l}\rangle$.

In the papers [11,13] it is proved, that for any changeable set $\mathcal{Z}$ the following properties are performed:

Properties 4. 1. *Any area of perception $\mathfrak{l} \in \mathcal{L}k(\mathcal{Z})$ can be represented in the form of ordered pair, $\mathfrak{l} = (\mathtt{ind}(\mathfrak{l}), \mathfrak{l}^{\wedge})$, where $\mathfrak{l}^{\wedge}$ is the base changeable set. Moreover: $\mathfrak{B}\mathfrak{s}(\mathfrak{l}) = \mathfrak{B}\mathfrak{s}(\mathfrak{l}^{\wedge})$, $\mathbb{B}\mathfrak{s}(\mathfrak{l}) = \mathbb{B}\mathfrak{s}(\mathfrak{l}^{\wedge})$, $\mathbf{Tm}(\mathfrak{l}) = \mathbf{Tm}(\mathfrak{l}^{\wedge})$, $\mathbb{T}\mathbf{m}(\mathfrak{l}) = \mathbb{T}\mathbf{m}(\mathfrak{l}^{\wedge})$, $\leq_{\mathfrak{l}} = \leq_{\mathfrak{l}^{\wedge}}$, $\underset{\mathfrak{l}}{\leftarrow} = \underset{\mathfrak{l}^{\wedge}}{\leftarrow}$, $\overset{\mathbb{B}\mathfrak{s}}{\underset{\mathfrak{l}}{\leftarrow}} = \overset{\mathbb{B}\mathfrak{s}}{\underset{\mathfrak{l}^{\wedge}}{\leftarrow}}$.*

2. *For arbitrary $\mathfrak{l} \in \mathcal{L}k(\mathcal{Z})$ and $A \subseteq \mathbb{B}\mathfrak{s}(\mathfrak{l})$ it is true the equality, $\langle \mathfrak{l} \leftarrow \mathfrak{l}\rangle A = A$.*
3. *If $\mathfrak{l}, \mathfrak{m} \in \mathcal{L}k(\mathcal{Z})$ and $A \subseteq B \subseteq \mathbb{B}\mathfrak{s}(\mathfrak{l})$, then $\langle \mathfrak{m} \leftarrow \mathfrak{l}\rangle A \subseteq \langle \mathfrak{m} \leftarrow \mathfrak{l}\rangle B$;*
4. *For arbitrary $\mathfrak{l}, \mathfrak{m}, \mathfrak{p} \in \mathcal{L}k(\mathcal{Z})$ and $A \subseteq \mathbb{B}\mathfrak{s}(\mathfrak{l})$ it holds the inclusion:*

$$\langle \mathfrak{p} \leftarrow \mathfrak{m}\rangle \langle \mathfrak{m} \leftarrow \mathfrak{l}\rangle A \subseteq \langle \mathfrak{p} \leftarrow \mathfrak{l}\rangle A.$$

Definition 1 ([17]). *Changeable set $\mathcal{Z}$ is named **precisely visible** if and only if for arbitrary $\mathfrak{l}, \mathfrak{m} \in \mathcal{L}k(\mathcal{Z})$ and $A \subseteq \mathbb{B}\mathfrak{s}(\mathfrak{l})$ the condition $A \neq \emptyset$ leads to $\langle \mathfrak{m} \leftarrow \mathfrak{l}\rangle A \neq \emptyset$.*

Assertion 2 ([11,12]). *Let $\mathfrak{l}, \mathfrak{m} \in \mathcal{L}k(\mathcal{Z})$ be arbitrary areas of perception of any precisely visible changeable set $\mathcal{Z}$. Then for any $\omega \in \mathbb{B}\mathfrak{s}(\mathfrak{l})$ the unique element $\omega' =: \langle !\, \mathfrak{m} \leftarrow \mathfrak{l}, \mathcal{Z}\rangle \omega \in \mathbb{B}\mathfrak{s}(\mathfrak{m})$ exists such, that $\{\omega'\} = \langle \mathfrak{m} \leftarrow \mathfrak{l}\rangle \{\omega\}$.*

Thus, in any precisely visible changeable set $\mathcal{Z}$ for arbitrary $\mathfrak{l}, \mathfrak{m} \in \mathcal{L}k(\mathcal{Z})$ and $\omega \in \mathbb{B}\mathfrak{s}(\mathfrak{l})$ it is performed the equality:

$$\{\langle !\, \mathfrak{m} \leftarrow \mathfrak{l}, \mathcal{Z}\rangle \omega\} = \langle \mathfrak{m} \leftarrow \mathfrak{l}\rangle \{\omega\}.$$

The mapping $\langle !\, \mathfrak{m} \leftarrow \mathfrak{l}, \mathcal{Z}\rangle : \mathbb{B}\mathfrak{s}(\mathfrak{l}) \mapsto \mathbb{B}\mathfrak{s}(\mathfrak{m})$ is referred to as ***precise unification*** mapping. In the cases, when the changeable set $\mathcal{Z}$ is known in advance, we use the denotation $\langle !\, \mathfrak{m} \leftarrow \mathfrak{l}\rangle$ instead of the denotation $\langle !\, \mathfrak{m} \leftarrow \mathfrak{l}, \mathcal{Z}\rangle$.

Assertion 3 ([11,12]). *Let $\mathfrak{l}, \mathfrak{m} \in \mathcal{L}k(\mathcal{Z})$, where $\mathcal{Z}$ is precisely visible changeable set. Then the sets $\mathbb{B}\mathfrak{s}(\mathfrak{l})$ and $\mathbb{B}\mathfrak{s}(\mathfrak{m})$ are equipotent. Moreover, the mapping: $f(\omega) = \langle !\, \mathfrak{m} \leftarrow \mathfrak{l}\rangle \omega$ ($\omega \in \mathbb{B}\mathfrak{s}(\mathfrak{l})$) is bijection (one-to-one correspondence) between these sets.*

Using Property 4(2), as well as [12, theorems 5.2, 5.1] or [11, theorems 11.2, 11.1], we receive the following properties of precise unification mappings in precisely visible changeable sets.

Properties 5. *Let $\mathcal{Z}$ be any precisely visible changeable set, and $\mathfrak{l}, \mathfrak{m}, \mathfrak{p} \in \mathcal{L}k(\mathcal{Z})$ be arbitrary areas of perception of $\mathcal{Z}$. Then:*

1. $\forall\, \omega \in \mathbb{Bs}(\mathfrak{l}) \quad \langle !\, \mathfrak{l} \leftarrow \mathfrak{l} \rangle\, \omega = \omega;$
2. $\forall\, A \subseteq \mathbb{Bs}(\mathfrak{l}) \quad \langle \mathfrak{m} \leftarrow \mathfrak{l} \rangle\, A = \{ \langle !\, \mathfrak{m} \leftarrow \mathfrak{l} \rangle\, \omega \mid \omega \in A \};$
3. $\forall\, \omega \in \mathbb{Bs}(\mathfrak{l}) \quad \langle !\, \mathfrak{p} \leftarrow \mathfrak{m} \rangle\, \langle !\, \mathfrak{m} \leftarrow \mathfrak{l} \rangle\, \omega = \langle !\, \mathfrak{p} \leftarrow \mathfrak{l} \rangle\, \omega.$

3 Changeable Sets and Kinematics.

3.1 Mathematical Objects for Constructing of Geometric Environments of Changeable Sets.

This subsection is purely technical in nature. In this subsection we don't introduce any essentially new notions. But we try to include the most frequently used mathematical spaces, which at least somehow related to geometry, into single mathematical structure, which will be convenient for further construction of abstract kinematics.

Definition 2. *The ordered triple $\mathbb{L} = (\mathbb{K}, \oplus, \otimes)$ will be named the **linear structure** over the set $\mathfrak{X}$, if and only if:*

1. $\mathbb{K} = (\mathbf{K}, +_{\mathbb{K}}, \times_{\mathbb{K}})$ *is a field.*
2. $\oplus : \mathfrak{X} \times \mathfrak{X} \mapsto \mathfrak{X}$ *is a binary operation over* $\mathfrak{X}$;
3. $\otimes : \mathbf{K} \times \mathfrak{X} \mapsto \mathfrak{X}$ *is a binary operation, acting from* $\mathbf{K} \times \mathfrak{X}$ *into* $\mathfrak{X}$.
4. *The ordered triple* $(\mathfrak{X}, \oplus, \otimes)$ *is a linear space over the field* $\mathbb{K}$.

*In the case, when $\mathbb{K} \in \{\mathbb{R}, \mathbb{C}\}$, the linear structure $\mathbb{L}$ will be named as **numerical linear structure** over $\mathfrak{X}$.*

Let $\mathbb{L} = (\mathbb{K}, \oplus, \otimes)$ be a linear structure over $\mathfrak{X}$. In this case the linear space over the field $\mathbb{K}$, generated by $\mathbb{L}$ will be denoted by $\mathfrak{Lp}(\mathfrak{X}, \mathbb{L}) \quad (\mathfrak{Lp}(\mathfrak{X}, \mathbb{L}) = (\mathfrak{X}, \oplus, \otimes))$.

The next definition is based on the conception, that the majority of the most frequently used mathematical objects (including functions, relations, algebraic operations, ordered pairs or compositions) are sets.

Definition 3. *An ordered composition of six sets $\mathfrak{Q} = (\mathfrak{X}, \mathcal{T}, \mathbb{L}, \rho, \|\cdot\|, (\cdot,\cdot))$ will be named as **coordinate space**, if and only if the following conditions are satisfied:*

1. $\mathfrak{X} \neq \emptyset$.
2. $\mathcal{T} \cup \mathbb{L} \neq \emptyset$.
3. *If* $\mathcal{T} \neq \emptyset$, *then* $\mathcal{T}$ *is topology over* $\mathfrak{X}$.
4. *If* $\mathbb{L} \neq \emptyset$, *then* $\mathbb{L}$ *is numerical linear structure over* $\mathfrak{X}$.
5. *If* $\mathbb{L} \neq \emptyset$ *and* $\mathcal{T} \neq \emptyset$, *then the pair* $(\mathfrak{Lp}(\mathfrak{X}, \mathbb{L}), \mathcal{T})$ *is a linear topological space.*
6. *If* $\rho \neq \emptyset$, *then:*

   6.1) ρ *is the metrics over* $\mathfrak{X}$;

   6.2) $\mathcal{T} \neq \emptyset$ *and the topology* $\mathcal{T}$ *is generated by the metrics* ρ.
7. *If* $\|\cdot\| \neq \emptyset$, *then:*

7.1) $\mathbb{L} \neq \emptyset$ *and* $\|\cdot\|$ *is the norm on the linear space* $\mathfrak{Lp}(\mathfrak{X}, \mathbb{L})$*;*

7.2) $\rho \neq \emptyset$ *and the metrics* ρ *is generated by the norm* $\|\cdot\|$*.*

8. *If* $(\cdot,\cdot) \neq \emptyset$*, then:*

8.1) $\|\cdot\| \neq \emptyset$ *(and hence, according to* **7.1)***,* $\mathbb{L} \neq \emptyset$*);*

8.2) $(\cdot,\cdot)$ *is the inner product on the linear space* $\mathfrak{Lp}(\mathfrak{X}, \mathbb{L})$*;*

8.3) *the norm* $\|\cdot\|$ *is generated by the inner product* $(\cdot,\cdot)$*.*

Notes on denotations. Let $\mathfrak{Q} = (\mathfrak{X}, \mathcal{T}, \mathbb{L}, \rho, \|\cdot\|, (\cdot,\cdot))$ be a coordinate space, where in the case $\mathbb{L} \neq \emptyset$ we have, that $\mathbb{L} = (\mathbb{K}, \oplus, \otimes)$ is a numerical linear structure over $\mathfrak{X}$. Further we will use the following denotations:

1. $\mathbf{Zk}(\mathfrak{Q}) := \mathfrak{X}$ (the set $\mathbf{Zk}(\mathfrak{Q})$ will be named the ***set of coordinate values*** of $\mathfrak{Q}$).
2. $\mathcal{T}p(\mathfrak{Q}) := \mathcal{T}$ ($\mathcal{T}p(\mathfrak{Q})$ will be referred to as ***topology*** of $\mathfrak{Q}$).
3. $\mathbb{L}s(\mathfrak{Q}) := \mathbb{L}$ ($\mathbb{L}s(\mathfrak{Q})$ will be named the ***linear structure*** of $\mathfrak{Q}$).
4. $\mathfrak{Ps}(\mathfrak{Q}) := \begin{cases} \mathbb{K}, & \mathbb{L}s(\mathfrak{Q}) \neq \emptyset \\ \emptyset, & \mathbb{L}s(\mathfrak{Q}) = \emptyset \end{cases}$ ($\mathfrak{Ps}(\mathfrak{Q})$ will be referred to as ***field of scalars*** of $\mathfrak{Q}$).
5. For the elements $x_1, \ldots, x_n \in \mathbf{Zk}(\mathfrak{Q})$, $\lambda_1, \ldots, \lambda_n \in \mathfrak{Ps}(\mathfrak{Q})$ ($n \in \mathbb{N}$) we will use the denotation, $(\lambda_1 x_1 + \cdots + \lambda_n x_n)_{\mathfrak{Q}} := \lambda_1 \otimes x_1 \oplus \cdots \oplus \lambda_n \otimes x_n$.
6. $\mathbf{di}_{\mathfrak{Q}} := \rho$ ($\mathbf{di}_{\mathfrak{Q}}$ will be named the ***distance*** on $\mathfrak{Q}$).
7. $\|\cdot\|_{\mathfrak{Q}} := \|\cdot\|$ ($\|\cdot\|_{\mathfrak{Q}}$ will be named the ***norm*** on $\mathfrak{Q}$).
8. $(\cdot,\cdot)_{\mathfrak{Q}} := (\cdot,\cdot)$ ($(\cdot,\cdot)_{\mathfrak{Q}}$ will be referred to as ***inner product*** on $\mathfrak{Q}$).

Elements of kind $x \in \mathbf{Zk}(\mathfrak{Q})$ will be named as ***coordinates*** of the coordinate space $\mathfrak{Q}$, also, in the case $\mathbb{L}s(\mathfrak{Q}) \neq \emptyset$ we will name these elements as ***vectors (vector coordinates)*** of $\mathfrak{Q}$. Where it does not cause confusion the symbol "$\mathfrak{Q}$" in the denotations $(\lambda_1 x_1 + \cdots + \lambda_n x_n)_{\mathfrak{Q}}$, $\mathbf{di}_{\mathfrak{Q}}$, $\|\cdot\|_{\mathfrak{Q}}$, $(\cdot,\cdot)_{\mathfrak{Q}}$ will be released, and we will use the abbreviated denotations $\lambda_1 x_1 + \cdots + \lambda_n x_n$, $\mathbf{di}$, $\|\cdot\|$, $(\cdot,\cdot)$ correspondingly.

3.2 Kinematic Changeable Sets.

Definition 4. *1. The pair* $\mathcal{G}_0 = (\mathfrak{Q}, k)$ *we name* ***geometric environment*** *of base changeable set* $\mathcal{B}$*, if and only if:*

a) $\mathfrak{Q}$ *is a coordinate space;*

b) $k : \mathfrak{Bs}(\mathcal{B}) \mapsto \mathbf{Zk}(\mathfrak{Q})$ *is a mapping from* $\mathfrak{Bs}(\mathcal{B})$ *into* $\mathbf{Zk}(\mathfrak{Q})$*.*

In this case the pair $\mathfrak{C}^{\mathfrak{b}} = (\mathcal{B}, \mathcal{G}_0) = (\mathcal{B}, (\mathfrak{Q}, k))$ *we name by* ***base kinematic changeable set****, or, abbreviated, by* ***base kinematic*** *set.*

2. Let $\mathcal{Z}$ *be any changeable set. An indexed family of pairs* $\mathcal{G} = ((\mathfrak{Q}_{\mathfrak{l}}, k_{\mathfrak{l}}) \mid \mathfrak{l} \in \mathcal{L}k(\mathcal{Z}))$ *will be named* ***geometric environment*** *of the changeable set* $\mathcal{Z}$*, if and only if for any area of perception* $\mathfrak{l} \in \mathcal{L}k(\mathcal{Z})$ *the ordered pair* $(\mathfrak{Q}_{\mathfrak{l}}, k_{\mathfrak{l}})$ *is geometric environment of the base changeable set* $\mathfrak{l}^{\wedge}$*, generated by the area of perception* $\mathfrak{l}$*, i.e. if and only if the pair* $(\mathfrak{l}^{\wedge}, (\mathfrak{Q}_{\mathfrak{l}}, k_{\mathfrak{l}}))$ *is a base kinematic changeable set for an arbitrary* $\mathfrak{l} \in \mathcal{L}k(\mathcal{Z})$*.*

In this case we name the pair $\mathfrak{C} = (\mathcal{Z}, \mathcal{G})$ *by* ***kinematic changeable set****, or, abbreviated, by* ***kinematic set****.*

Note, that in this article we consider only kinematic sets with constant (unchanging over time) geometry. These kinematic sets are sufficient for construction of abstract kinematics in inertial reference frames. If we make a some modification of the Definition 4, we will be able to define also kinematic sets with variable (over time) geometry (i.e., in principle, this is, possible to do).

3.2.1 System of Denotations for Base Kinematic Sets.

Let, $\mathfrak{C}^{\mathfrak{b}} = (\mathcal{B}, \mathcal{G}_0)$ be any base kinematic set (where $\mathcal{G}_0 = (\mathfrak{Q}, k)$). Henceforth we use the following system of denotations.

a) Denotations, induced from the theory of base changeable sets:

$$\begin{array}{ll} \mathfrak{Bs}\left(\mathfrak{C}^{\mathfrak{b}}\right) := \mathfrak{Bs}(\mathcal{B}); & \mathbb{B}\mathfrak{s}\left(\mathfrak{C}^{\mathfrak{b}}\right) := \mathbb{B}\mathfrak{s}(\mathcal{B}); \\ \underset{\mathfrak{C}^{\mathfrak{b}}}{\leftarrow} := \underset{\mathcal{B}}{\leftarrow}; & \overset{\mathbb{B}\mathfrak{s}}{\underset{\mathfrak{C}^{\mathfrak{b}}}{\leftarrow}} := \overset{\mathbb{B}\mathfrak{s}}{\underset{\mathcal{B}}{\leftarrow}}; \\ \mathbf{Tm}\left(\mathfrak{C}^{\mathfrak{b}}\right) := \mathbf{Tm}(\mathcal{B}); & \mathbb{T}\mathbf{m}\left(\mathfrak{C}^{\mathfrak{b}}\right) := \mathbb{T}\mathbf{m}(\mathcal{B}); \quad \leq_{\mathfrak{C}^{\mathfrak{b}}} := \leq_{\mathcal{B}} . \end{array}$$

b) Denotations, induced from the denotations for coordinate spaces:

$$\begin{array}{lll} \mathbf{Zk}\left(\mathfrak{C}^{\mathfrak{b}}\right) := \mathbf{Zk}\left(\mathfrak{Q}\right); & \mathcal{T}p\left(\mathfrak{C}^{\mathfrak{b}}\right) := \mathcal{T}p\left(\mathfrak{Q}\right); & \mathbb{L}s\left(\mathfrak{C}^{\mathfrak{b}}\right) := \mathbb{L}s\left(\mathfrak{Q}\right); \\ \mathfrak{Ps}\left(\mathfrak{C}^{\mathfrak{b}}\right) := \mathfrak{Ps}\left(\mathfrak{Q}\right); & \mathbf{di}_{\mathfrak{C}^{\mathfrak{b}}} := \mathbf{di}_{\mathfrak{Q}}; & \|\cdot\|_{\mathfrak{C}^{\mathfrak{b}}} := \|\cdot\|_{\mathfrak{Q}}; \\ (\cdot,\cdot)_{\mathfrak{C}^{\mathfrak{b}}} := (\cdot,\cdot)_{\mathfrak{Q}}. & & \end{array}$$

Also in the case $\mathbb{L}s\left(\mathfrak{C}^{\mathfrak{b}}\right) \neq \emptyset$ for arbitrary $a_1, \ldots, a_n \in \mathbf{Zk}\left(\mathfrak{C}^{\mathfrak{b}}\right)$, $\lambda_1, \ldots \lambda_n \in \mathfrak{Ps}\left(\mathfrak{C}^{\mathfrak{b}}\right)$ we use the denotation, $(\lambda_1 a_1 + \cdots + \lambda_n a_n)_{\mathfrak{C}^{\mathfrak{b}}} := (\lambda_1 a_1 + \cdots + \lambda_n a_n)_{\mathcal{Q}}$.

c) Own designations for base kinematic sets:

$$\mathsf{BE}\left(\mathfrak{C}^{\mathfrak{b}}\right) := \mathcal{B}; \qquad \mathsf{BG}\left(\mathfrak{C}^{\mathfrak{b}}\right) := \mathfrak{Q}; \qquad \mathfrak{q}_{\mathfrak{C}^{\mathfrak{b}}}(x) := k(x) \ \left(x \in \mathfrak{Bs}\left(\mathfrak{C}^{\mathfrak{b}}\right)\right).$$

Note, that for any elementary state $x \in \mathfrak{Bs}\left(\mathfrak{C}^{\mathfrak{b}}\right)$ the function $\mathfrak{q}_{\mathfrak{C}^{\mathfrak{b}}}(\cdot)$ puts in accordance its coordinate $\mathfrak{q}_{\mathfrak{C}^{\mathfrak{b}}}(x) \in \mathbf{Zk}\left(\mathfrak{C}^{\mathfrak{b}}\right)$.

d) Abbreviated version of denotations

- We use all abbreviated variants of denotations, described in the Remark 1 (but, with the replacement of the symbol "$\mathcal{B}$" by the symbol "$\mathfrak{C}^{\mathfrak{b}}$" and the term "base changeable set" by the term "base kinematic set".
- In the cases, when the base kinematic set $\mathfrak{C}^{\mathfrak{b}}$ is known in advance, we will use the denotations $\mathbf{di}$, $\|\cdot\|$, $(\cdot,\cdot)$, $\mathfrak{q}(x)$ instead of the denotations $\mathbf{di}_{\mathfrak{C}^{\mathfrak{b}}}$, $\|\cdot\|_{\mathfrak{C}^{\mathfrak{b}}}$, $(\cdot,\cdot)_{\mathfrak{C}^{\mathfrak{b}}}$, $\mathfrak{q}_{\mathfrak{C}^{\mathfrak{b}}}(x)$ (correspondingly).

3.2.2 System of Denotations for Kinematic Sets.

Let, $\mathfrak{C} = (\mathcal{Z}, \mathcal{G})$, where $\mathcal{G} = ((\mathfrak{Q}_{\mathfrak{l}}, k_{\mathfrak{l}}) \mid \mathfrak{l} \in \mathcal{L}k\left(\mathcal{Z}\right))$ be any kinematic set.

a) The changeable set $\mathbb{BE}\left(\mathfrak{C}\right) := \mathcal{Z}$ will be named the ***evolution base*** of the kinematic set $\mathfrak{C}$.

b) The sets $\mathcal{I}nd\left(\mathfrak{C}\right) := \mathcal{I}nd\left(\mathcal{Z}\right) = \mathcal{I}nd\left(\mathbb{BE}\left(\mathfrak{C}\right)\right)$; $\mathcal{L}k\left(\mathfrak{C}\right) := \mathcal{L}k\left(\mathcal{Z}\right) = \mathcal{L}k\left(\mathbb{BE}\left(\mathfrak{C}\right)\right)$ will be named the set of ***indexes*** and the the set of all ***reference frames*** of kinematic set $\mathfrak{C}$ (correspondingly). Note, that in the case of kinematic sets (in contrast to the case of changeable sets) we use the term "reference frame" instead of the term "area of perception".

c) For any reference frame $\mathfrak{l} \in \mathcal{L}k\left(\mathfrak{C}\right) = \mathcal{L}k\left(\mathcal{Z}\right)$ we keep all denotations, introduced for area of perception of changeable sets (it concerns the denotations: $\mathtt{ind}\left(\mathfrak{l}\right)$, $\mathfrak{Bs}(\mathfrak{l})$, $\underset{\mathfrak{l}}{\leftarrow}$, $\mathbb{B}\mathfrak{s}(\mathfrak{l})$, $\overset{\mathbb{B}\mathfrak{s}}{\underset{\mathfrak{l}}{\leftarrow}}$, $\mathbf{Tm}(\mathfrak{l})$, $\mathbb{T}\mathbf{m}(\mathfrak{l})$, $\leq_{\mathfrak{l}}$).

d) For arbitrary reference frames $\mathfrak{l}, \mathfrak{m} \in \mathcal{L}k(\mathfrak{C})$ it is induced the denotation for unification mapping:

$$\langle \mathfrak{m} \leftarrow \mathfrak{l},\ \mathfrak{C} \rangle := \langle \mathfrak{m} \leftarrow \mathfrak{l},\ \mathcal{Z} \rangle\,.$$

In particular in the case, when the changeable set $\mathcal{Z}$ is precisely visible (in this case we say, that the kinematic set $\mathfrak{C}$ is ***precisely visible***), we introduce the denotation:

$$\langle !\,\mathfrak{m} \leftarrow \mathfrak{l}, \mathfrak{C} \rangle\ := \langle !\,\mathfrak{m} \leftarrow \mathfrak{l}, \mathcal{Z} \rangle\,.$$

e) For any reference frame $\mathfrak{l} \in \mathcal{L}k(\mathfrak{C})$ we introduce the denotation $\mathfrak{C} \upharpoonright \mathfrak{l} = (\mathfrak{l}^{\hat{}}, (\mathfrak{Q}_\mathfrak{l}, k_\mathfrak{l}))$. By the Definition 4, the pair $\mathfrak{C} \upharpoonright \mathfrak{l}$ is a base kinematic set (for arbitrary reference frame $\mathfrak{l} \in \mathcal{L}k(\mathfrak{C})$). The base kinematic set $\mathfrak{C} \upharpoonright \mathfrak{l}$ will be named the ***image of kinematic set*** $\mathfrak{C}$ in the reference frame $\mathfrak{l}$.

f) For any reference frame $\mathfrak{l} \in \mathcal{L}k(\mathfrak{C})$ we introduce the following denotations:

$$\begin{array}{ll}
\mathbf{Zk}(\mathfrak{l};\,\mathfrak{C}) := \mathbf{Zk}(\mathfrak{C} \upharpoonright \mathfrak{l}) = \mathbf{Zk}(\mathfrak{Q}_\mathfrak{l})\,; & \mathbb{L}s(\mathfrak{l};\,\mathfrak{C}) := \mathbb{L}s(\mathfrak{C} \upharpoonright \mathfrak{l}) = \mathbb{L}s(\mathfrak{Q}_\mathfrak{l})\,;\\
\mathcal{T}p(\mathfrak{l};\,\mathfrak{C}) := \mathcal{T}p(\mathfrak{C} \upharpoonright \mathfrak{l}) = \mathcal{T}p(\mathfrak{Q}_\mathfrak{l})\,; & \mathfrak{Ps}(\mathfrak{l};\,\mathfrak{C}) := \mathfrak{Ps}(\mathfrak{C} \upharpoonright \mathfrak{l}) = \mathfrak{Ps}(\mathfrak{Q}_\mathfrak{l})\,;\\
\|\cdot\|_{\mathfrak{l},\mathfrak{C}} := \|\cdot\|_{\mathfrak{C}\upharpoonright\mathfrak{l}} = \|\cdot\|_{\mathfrak{Q}_\mathfrak{l}}\,; & \mathbf{di}_\mathfrak{l}(\cdot;\,\mathfrak{C}) := \mathbf{di}_{\mathfrak{C}\upharpoonright\mathfrak{l}} = \mathbf{di}_{\mathfrak{Q}_\mathfrak{l}};\\
(\cdot,\cdot)_{\mathfrak{l},\mathfrak{C}} := (\cdot,\cdot)_{\mathfrak{C}\upharpoonright\mathfrak{l}} = (\cdot,\cdot)_{\mathfrak{Q}_\mathfrak{l}}\,; & \mathsf{BE}(\mathfrak{l}) := \mathsf{BE}(\mathfrak{C} \upharpoonright \mathfrak{l}) = \mathfrak{l}^{\hat{}};\\
 & \mathsf{BG}(\mathfrak{l};\,\mathfrak{C}) := \mathsf{BG}(\mathfrak{C} \upharpoonright \mathfrak{l}) = \mathfrak{Q}_\mathfrak{l}.
\end{array}$$

Also for reference frames $\mathfrak{l} \in \mathcal{L}k(\mathfrak{C})$ such, that $\mathbb{L}s(\mathfrak{l}) \neq \emptyset$ and for arbitrary $a_1, \ldots, a_n \in \mathbf{Zk}(\mathfrak{l};\,\mathfrak{C})$, $\lambda_1, \ldots \lambda_n \in \mathfrak{Ps}(\mathfrak{l};\,\mathfrak{C})$ we will use the denotation, $(\lambda_1 a_1 + \cdots + \lambda_n a_n)_{\mathfrak{l},\mathfrak{C}} := (\lambda_1 a_1 + \cdots + \lambda_n a_n)_{\mathcal{Q}_l}$.

g) For any reference frame $\mathfrak{l} \in \mathcal{L}k(\mathfrak{C})$ we use the following denotation:

$$\mathfrak{q}_\mathfrak{l}(x;\,\mathfrak{C}) := \mathfrak{q}_{\mathfrak{C}\upharpoonright\mathfrak{l}}(x) = k_\mathfrak{l}(x),\ x \in \mathfrak{Bs}(\mathfrak{l})\,.$$

h) Abbreviated versions of denotations:

- In the cases, when the kinematic set $\mathfrak{C}$ is known in advance, we will use the denotations $\langle \mathfrak{m} \leftarrow \mathfrak{l} \rangle$, $\langle !\,\mathfrak{m} \leftarrow \mathfrak{l} \rangle$, $\mathbf{Zk}(\mathfrak{l})$, $\mathbb{L}s(\mathfrak{l})$, $\mathbf{di}_\mathfrak{l}$, $(\cdot,\cdot)_\mathfrak{l}$, $\mathcal{T}p(\mathfrak{l})$, $\mathfrak{Ps}(\mathfrak{l})$, $\|\cdot\|_\mathfrak{l}$, $\mathsf{BG}(\mathfrak{l})$, $\mathfrak{q}_l(x)$ instead of the denotations $\langle \mathfrak{m} \leftarrow \mathfrak{l},\ \mathfrak{C} \rangle$, $\langle !\,\mathfrak{m} \leftarrow \mathfrak{l},\ \mathfrak{C} \rangle$, $\mathbf{Zk}(\mathfrak{l};\,\mathfrak{C})$, $\mathbb{L}s(\mathfrak{l};\,\mathfrak{C})$, $\mathbf{di}_\mathfrak{l}(\cdot;\,\mathfrak{C})$, $(\cdot,\cdot)_{\mathfrak{l},\mathfrak{C}}$, $\mathcal{T}p(\mathfrak{l};\,\mathfrak{C})$, $\mathfrak{Ps}(\mathfrak{l};\,\mathfrak{C})$, $\|\cdot\|_{\mathfrak{l},\mathfrak{C}}$, $\mathsf{BG}(\mathfrak{l};\,\mathfrak{C})$, $\mathfrak{q}_l(x;\,\mathfrak{C})$ (correspondingly).
- In the cases, when the reference frame $\mathfrak{l} \in \mathcal{L}k(\mathfrak{C})$ is known in advance, we will use the denotations $\mathbf{di}$, $\|\cdot\|$, $(\cdot,\cdot)$, $\mathfrak{q}(x)$, $\lambda_1 a_1 + \cdots + \lambda_n a_n$ instead of the denotations $\mathbf{di}_\mathfrak{l}$, $\|\cdot\|_\mathfrak{l}$, $(\cdot,\cdot)_\mathfrak{l}$, $\mathfrak{q}_\mathfrak{l}(x)$, $(\lambda_1 a_1 + \cdots + \lambda_n a_n)_{\mathfrak{l},\mathfrak{C}}$ (correspondingly). Also we use all abbreviated variants of denotations for areas of perception of changeable sets, described in the Remark 3.

Assertion 4 ([17])**.** *Let $\mathfrak{C}_1$, $\mathfrak{C}_2$ be arbitrary kinematic sets, and besides:*

1. $\mathcal{L}k(\mathfrak{C}_1) = \mathcal{L}k(\mathfrak{C}_2)$.
2. *For any reference frame $\mathfrak{l} \in \mathcal{L}k(\mathfrak{C}_1) = \mathcal{L}k(\mathfrak{C}_2)$ they are true the equalities,* $\mathsf{BG}(\mathfrak{l};\,\mathfrak{C}_1) = \mathsf{BG}(\mathfrak{l};\,\mathfrak{C}_2)$ *and* $\mathfrak{q}_\mathfrak{l}(x,\,\mathfrak{C}_1) = \mathfrak{q}_\mathfrak{l}(x,\,\mathfrak{C}_2)$ $(\forall x \in \mathfrak{Bs}(\mathfrak{l}))$.
3. *For arbitrary reference frames $\mathfrak{l}, \mathfrak{m} \in \mathcal{L}k(\mathfrak{C}_1) = \mathcal{L}k(\mathfrak{C}_2)$ it is true the equality,* $\langle \mathfrak{m} \leftarrow \mathfrak{l}, \mathfrak{C}_1 \rangle = \langle \mathfrak{m} \leftarrow \mathfrak{l}, \mathfrak{C}_2 \rangle$.

Then, $\mathfrak{C}_1 = \mathfrak{C}_2$.

Remark 4. From the system of denotations, accepted in the subsection 3.2, it follows, that for any kinematic set $\mathfrak{C}$, Properties 3, Properties 4 and Assertion 3 are kept to be true, and in the case, when the kinematic set $\mathfrak{C}$ is precisely visible, Properties 5 also remain true (but everywhere in these properties we should replace the symbol $\mathcal{Z}$ by the symbol $\mathfrak{C}$ and the terms "changeable set" and "area of perception" by the terms "kinematic set" and "reference frame", correspondingly).

4 Coordinate Transforms in Kinematic Sets.

Let, $\mathfrak{C}$ be any kinematic set. For any reference frame $\mathfrak{l} \in \mathcal{L}k(\mathfrak{C})$ we introduce the following denotations:

$$\mathbb{M}k(\mathfrak{l};\mathfrak{C}) := \mathbf{Tm}(\mathfrak{l}) \times \mathbf{Zk}(\mathfrak{l}).$$
$$\mathbf{Q}^{\langle \mathfrak{l} \rangle}(\omega;\mathfrak{C}) := (\mathsf{tm}(\omega), \mathsf{q}_{\mathfrak{l}}(\mathsf{bs}(\omega))) \in \mathbb{M}k(\mathfrak{l};\mathfrak{C}), \quad \omega \in \mathbb{Bs}(\mathfrak{l}).$$

The set $\mathbb{M}k(\mathfrak{l};\mathfrak{C})$ we name as the ***Minkowski set*** of reference frame $\mathfrak{l}$ in kinematic set $\mathfrak{C}$. The value $\mathbf{Q}^{\langle \mathfrak{l} \rangle}(\omega;\mathfrak{C})$ will be named as the ***Minkowski coordinates*** of the elementary-time state $\omega \in \mathbb{Bs}(\mathfrak{l})$ ***in the reference frame*** $\mathfrak{l}$.

In the cases, when the kinematic set $\mathfrak{C}$ is known in advance, we use the denotations $\mathbb{M}k(\mathfrak{l})$, $\mathbf{Q}^{\langle \mathfrak{l} \rangle}(\omega)$ instead of the denotations $\mathbb{M}k(\mathfrak{l};\mathfrak{C})$, $\mathbf{Q}^{\langle \mathfrak{l} \rangle}(\omega;\mathfrak{C})$ (correspondingly).

Definition 5. *Let $\mathfrak{C}$ be any precisely visible kinematic set and $\mathfrak{l},\mathfrak{m} \in \mathcal{L}k(\mathfrak{C})$ be arbitrary reference frames of $\mathfrak{C}$.*

1. *The mapping $\mathbf{Q}^{\langle \mathfrak{m} \leftarrow \mathfrak{l} \rangle}(\,\cdot\,;\mathfrak{C}) : \mathbb{Bs}(\mathfrak{l}) \mapsto \mathbb{M}k(\mathfrak{m})$, represented by the formula:*

   $$\mathbf{Q}^{\langle \mathfrak{m} \leftarrow \mathfrak{l} \rangle}(\omega;\mathfrak{C}) = \mathbf{Q}^{\langle \mathfrak{m} \rangle}(\langle !\, \mathfrak{m} \leftarrow \mathfrak{l} \rangle\, \omega), \quad \omega \in \mathbb{Bs}(\mathfrak{l})$$

   *we name **actual coordinate transform** from $\mathfrak{l}$ to $\mathfrak{m}$.*
   Hence, for any $\omega \in \mathbb{Bs}(\mathfrak{l})$ the value $\mathbf{Q}^{\langle \mathfrak{m} \leftarrow \mathfrak{l} \rangle}(\omega;\mathfrak{C})$ may be interpreted as Minkowski coordinates of the elementary-time state ω in the (another) reference frame $\mathfrak{m} \in \mathcal{L}k(\mathfrak{C})$.

2. *The mapping $\widetilde{Q} : \mathbb{M}k(\mathfrak{l}) \mapsto \mathbb{M}k(\mathfrak{m})$ we will name the **universal coordinate transform** from $\mathfrak{l}$ to $\mathfrak{m}$ if and only if:*

   - *$\widetilde{Q}$ is bijection (one-to-one mapping) from $\mathbb{M}k(\mathfrak{l})$ onto $\mathbb{M}k(\mathfrak{m})$.*
   - *For any elementary-time state $\omega \in \mathbb{Bs}(\mathfrak{l})$ the following equality is performed:*

   $$\mathbf{Q}^{\langle \mathfrak{m} \leftarrow \mathfrak{l} \rangle}(\omega;\mathfrak{C}) = \widetilde{Q}\left(\mathbf{Q}^{\langle \mathfrak{l} \rangle}(\omega)\right).$$

3. *We say, that reference frames $\mathfrak{l},\mathfrak{m} \in \mathcal{L}k(\mathfrak{C})$ **allow universal coordinate transform**, if and only if at least one universal coordinate transform $\widetilde{Q} : \mathbb{M}k(\mathfrak{l}) \mapsto \mathbb{M}k(\mathfrak{m})$ from $\mathfrak{l}$ to $\mathfrak{m}$ exists.*
   In the case, where reference frames $\mathfrak{l},\mathfrak{m} \in \mathcal{L}k(\mathfrak{C})$ allow universal coordinate transform, we use the denotation:

   $$\mathfrak{l} \underset{\mathfrak{C}}{\rightleftarrows} \mathfrak{m},$$

   In the case, when the kinematic set $\mathfrak{C}$ is known in advance, we use the abbreviated denotation $\mathfrak{l} \rightleftarrows \mathfrak{m}$.

4. *Indexed family of mappings $\left(\widetilde{Q}_{\mathfrak{m},\mathfrak{l}}\right)_{\mathfrak{l},\mathfrak{m} \in \mathcal{L}k(\mathfrak{C})}$ will be named as the **universal coordinate transform for the kinematic set** $\mathfrak{C}$ if and only if:*

   - *For arbitrary $\mathfrak{l},\mathfrak{m} \in \mathcal{L}k(\mathfrak{C})$ the mapping $\widetilde{Q}_{\mathfrak{m},\mathfrak{l}}$ is universal coordinate transform from $\mathfrak{l}$ to $\mathfrak{m}$.*
   - *For any $\mathfrak{l},\mathfrak{m},\mathfrak{p} \in \mathcal{L}k(\mathfrak{C})$ and $\mathrm{w} \in \mathbb{M}k(\mathfrak{l})$ the following equalities are true:*

   $$\widetilde{Q}_{\mathfrak{l},\mathfrak{l}}(\mathrm{w}) = \mathrm{w}; \qquad \widetilde{Q}_{\mathfrak{p},\mathfrak{m}}\left(\widetilde{Q}_{\mathfrak{m},\mathfrak{l}}(\mathrm{w})\right) = \widetilde{Q}_{\mathfrak{p},\mathfrak{l}}(\mathrm{w}). \tag{2}$$

5. *We say, that the kinematic set $\mathfrak{C}$ **allows universal coordinate transform**, if and only if there exists at least one universal coordinate transform $\left(\widetilde{Q}_{\mathfrak{m},\mathfrak{l}}\right)_{\mathfrak{l},\mathfrak{m} \in \mathcal{L}k(\mathfrak{C})}$ for $\mathfrak{C}$.*

Remark 5. In the cases, when the kinematic set $\mathfrak{C}$ is known in advance, we use the abbreviated denotation $\mathbf{Q}^{\langle \mathfrak{m} \leftarrow \mathfrak{l} \rangle}(\omega)$ instead of the denotation $\mathbf{Q}^{\langle \mathfrak{m} \leftarrow \mathfrak{l} \rangle}(\omega; \mathfrak{C})$.

Assertion 5 ([17]). *Let $\mathfrak{C}$ be any precisely visible kinematic set. Then:*

1. *For an arbitrary $\mathfrak{l} \in \mathcal{L}k(\mathfrak{C})$ the identity mapping on $\mathbb{M}k(\mathfrak{l})$:*

$$\mathbb{I}_{[\mathfrak{l}]}(\mathrm{w}) := \mathrm{w}, \qquad \mathrm{w} \in \mathbb{M}k(\mathfrak{l})$$

   is universal coordinate transform from $\mathfrak{l}$ to $\mathfrak{l}$.

2. *If $\widetilde{Q}$ is universal coordinate transform from $\mathfrak{l}$ to $\mathfrak{m}$ ($\mathfrak{l}, \mathfrak{m} \in \mathcal{L}k(\mathfrak{C})$), then $\widetilde{Q}^{[-1]}$ is universal coordinate transform from $\mathfrak{m}$ to $\mathfrak{l}$ (the mapping $\widetilde{Q}^{[-1]}$, inverse to $\widetilde{Q}$, exists, because, according to the Definition 5 (item 2), $\widetilde{Q}$ is bijection from $\mathbb{M}k(\mathfrak{l})$ on $\mathbb{M}k(\mathfrak{m})$).*

3. *If $\widetilde{Q}^{(\mathfrak{m},\mathfrak{l})}$ is universal coordinate transform from $\mathfrak{l}$ to $\mathfrak{m}$, and $\widetilde{Q}^{(\mathfrak{p},\mathfrak{m})}$ is universal coordinate transform from $\mathfrak{m}$ to $\mathfrak{p}$ ($\mathfrak{l}, \mathfrak{m}, \mathfrak{p} \in \mathcal{L}k(\mathfrak{C})$), then the composition of the mappings $\widetilde{Q}^{(\mathfrak{p},\mathfrak{m})}$ and $\widetilde{Q}^{(\mathfrak{m},\mathfrak{l})}$, that is the mapping:*

$$\widetilde{Q}^{(\mathfrak{p},\mathfrak{l})}(\mathrm{w}) = \widetilde{Q}^{(\mathfrak{p},\mathfrak{m})}\left(\widetilde{Q}^{(\mathfrak{m},\mathfrak{l})}(\mathrm{w})\right), \quad \mathrm{w} \in \mathbb{M}k(\mathfrak{l}).$$

   is universal coordinate transform from $\mathfrak{l}$ to $\mathfrak{p}$.

4. *The binary relation $\rightleftarrows$ is equivalence relation on the set $\mathcal{L}k(\mathfrak{C})$ of all reference frames of $\mathfrak{C}$.*

Assertion 6 ([17]). *For an arbitrary precisely visible kinematic set $\mathfrak{C}$ the following propositions are equivalent:*

1. *$\mathfrak{C}$ allows universal coordinate transform.*

2. *For arbitrary reference frames $\mathfrak{l}, \mathfrak{m} \in \mathcal{L}k(\mathfrak{C})$ it is true the correlation $\mathfrak{l} \rightleftarrows \mathfrak{m}$ (that is arbitrary two reference frames $\mathfrak{l}, \mathfrak{m} \in \mathcal{L}k(\mathfrak{C})$ allow universal coordinate transform).*

3. *There exists a reference frame $\mathfrak{l} \in \mathcal{L}k(\mathfrak{C})$ such, that for any reference frame $\mathfrak{m} \in \mathcal{L}k(\mathfrak{C})$ it is true the correlation $\mathfrak{l} \rightleftarrows \mathfrak{m}$.*

Let $\mathfrak{C}$ be any kinematic set. For arbitrary reference frame $\mathfrak{l} \in \mathcal{L}k(\mathfrak{C})$ we denote:

$$\begin{aligned} &\mathbb{T}\mathbf{rj}(\mathfrak{l};\, \mathfrak{C}) := \left\{ \mathbf{Q}^{\langle \mathfrak{l} \rangle}(\omega) \mid \omega \in \mathbb{B}\mathfrak{s}(\mathfrak{l}) \right\}; \\ &\overline{\mathbb{T}\mathbf{rj}}(\mathfrak{l};\, \mathfrak{C}) := \mathbb{M}k(\mathfrak{l}) \setminus \mathbb{T}\mathbf{rj}(\mathfrak{l};\, \mathfrak{C}) \end{aligned} \tag{3}$$

(In the cases, when the kinematic set $\mathfrak{C}$ is known in advance, we use the abbreviated denotations $\mathbb{T}\mathbf{rj}(\mathfrak{l})$, $\overline{\mathbb{T}\mathbf{rj}}(\mathfrak{l})$ instead of the denotations $\mathbb{T}\mathbf{rj}(\mathfrak{l};\, \mathfrak{C})$, $\overline{\mathbb{T}\mathbf{rj}}(\mathfrak{l};\, \mathfrak{C})$ (correspondingly).) The set $\mathbb{T}\mathbf{rj}(\mathfrak{l})$ will be named the ***(general) trajectory*** for the reference frame $\mathfrak{l}$, and the set $\overline{\mathbb{T}\mathbf{rj}}(\mathfrak{l})$ will be named as ***complement of (general) trajectory*** of the reference frame $\mathfrak{l}$ in the kinematic set $\mathfrak{C}$.

Theorem 1 ([17]). *Let $\mathfrak{C}$ be a precisely visible kinematic set and $\mathfrak{l}, \mathfrak{m} \in \mathcal{L}k(\mathfrak{C})$ be any fixed reference frames of $\mathfrak{C}$.*

The reference frames $\mathfrak{l}, \mathfrak{m}$ allow universal coordinate transform (i.e. $\mathfrak{l} \rightleftarrows \mathfrak{m}$) if and only if the following conditions are satisfied:

1. $\mathbf{card}\left(\overline{\mathbb{T}\mathbf{rj}}(\mathfrak{l})\right) = \mathbf{card}\left(\overline{\mathbb{T}\mathbf{rj}}(\mathfrak{m})\right)$, *where* $\mathbf{card}(\mathcal{M})$ *means the* ***cardinality*** *of a set $\mathcal{M}$.*

2. *For arbitrary elementary-time states $\omega_1, \omega_2 \in \mathbb{B}\mathfrak{s}(\mathfrak{l})$ the equality $\mathbf{Q}^{\langle \mathfrak{m} \leftarrow \mathfrak{l} \rangle}(\omega_1) = \mathbf{Q}^{\langle \mathfrak{m} \leftarrow \mathfrak{l} \rangle}(\omega_2)$ is performed if and only if $\mathbf{Q}^{\langle \mathfrak{l} \rangle}(\omega_1) = \mathbf{Q}^{\langle \mathfrak{l} \rangle}(\omega_2)$.*

5 Theorems on Multi-image.

In this section we prove the theorems on multi-image, necessary to build mathematically strict model of kinematics of special relativity and its extension to the kinematics that allows super-light motion for inertial reference frames.

5.1 Theorem on Image for Base Changeable Sets.

Definition 6. *The ordered triple* $(\mathbb{T}, \mathcal{X}, U)$ *will be referred to as* ***evolution projector*** *for base changeable set* $\mathcal{B}$ *if and only if:*

1. $\mathbb{T} = (\mathbf{T}, \leq)$ *is linearly ordered set;*
2. $\mathcal{X}$ *is any set;*
3. U *is a mapping from* $\mathbb{B}\mathfrak{s}(\mathcal{B})$ *into* $\mathbf{T} \times \mathcal{X}$ $(U : \mathbb{B}\mathfrak{s}(\mathcal{B}) \mapsto \mathbf{T} \times \mathcal{X})$.

Definition 7 ([12]). *Let* $\mathcal{B}$ *be any base changeable set. We will say, that elementary-time states* $\omega_1, \omega_2 \in \mathbb{B}\mathfrak{s}(\mathcal{B})$ *are* **united by fate** *in* $\mathcal{B}$ *if and only if at least one of the conditions* $\omega_2 \leftarrow \omega_1$ *or* $\omega_1 \leftarrow \omega_2$ *is satisfied.*

Theorem 2 ([11, 13]). *Let* $(\mathbb{T}, \mathcal{X}, U)$ *be any evolution projector for base changeable set* $\mathcal{B}$*. Then there exist only one base changeable set* $U[\mathcal{B}, \mathbb{T}]$*, satisfying the following conditions:*

1. $\mathbb{T}\mathbf{m}\,(U[\mathcal{B}, \mathbb{T}]) = \mathbb{T}$*;*
2. $\mathbb{B}\mathfrak{s}(U[\mathcal{B}, \mathbb{T}]) = U(\mathbb{B}\mathfrak{s}(\mathcal{B})) = \{U(\omega) \mid \omega \in \mathbb{B}\mathfrak{s}(\mathcal{B})\}$*;*
3. *Let* $\widetilde{\omega}_1, \widetilde{\omega}_2 \in \mathbb{B}\mathfrak{s}(U[\mathcal{B}, \mathbb{T}])$ *and* $\mathsf{tm}(\widetilde{\omega}_1) \neq \mathsf{tm}(\widetilde{\omega}_2)$*. Then* $\widetilde{\omega}_1$ *and* $\widetilde{\omega}_2$ *are united by fate in* $U[\mathcal{B}, \mathbb{T}]$ *if and only if, there exist united by fate in* $\mathcal{B}$ *elementary-time states* $\omega_1, \omega_2 \in \mathbb{B}\mathfrak{s}(\mathcal{B})$ *such, that* $\widetilde{\omega}_1 = U(\omega_1)$*,* $\widetilde{\omega}_2 = U(\omega_2)$*.*

Remark 6. In the case, when $\mathbb{T} = \mathbb{T}\mathbf{m}(\mathcal{B})$ we use the denotation $U[\mathcal{B}]$ instead of the denotation $U[\mathcal{B}, \mathbb{T}]$:

$$U[\mathcal{B}] := U[\mathcal{B}, \mathbb{T}\mathbf{m}(\mathcal{B})]\,.$$

Remark 7. Let $\mathcal{B}$ be any base changeable set and $\mathbb{I}_{\mathbb{B}\mathfrak{s}(\mathcal{B})} : \mathbb{B}\mathfrak{s}(\mathcal{B}) \mapsto \mathbf{Tm}(\mathcal{B}) \times \mathfrak{B}\mathfrak{s}(\mathcal{B})$ be the mapping, given by the formula: $\mathbb{I}_{\mathbb{B}\mathfrak{s}(\mathcal{B})}(\omega) = \omega \quad (\omega \in \mathbb{B}\mathfrak{s}(\mathcal{B}))$. Then the triple $\left(\mathbb{T}\mathbf{m}(\mathcal{B}), \mathfrak{B}\mathfrak{s}(\mathcal{B}), \mathbb{I}_{\mathbb{B}\mathfrak{s}(\mathcal{B})}\right)$, is, apparently, evolution projector for $\mathcal{B}$. Moreover, if we substitute $\mathbb{T}\mathbf{m}(\mathcal{B})$ and $\mathcal{B}$ into Theorem 2 instead of $\mathbb{T}$ and $U[\mathcal{B}, \mathbb{T}]$ (correspondingly), we can see, that all conditions of this Theorem are satisfied. Hence for the identity mapping $\mathbb{I}_{\mathbb{B}\mathfrak{s}(\mathcal{B})}$ (on $\mathbb{B}\mathfrak{s}(\mathcal{B})$), we obtain:

$$\mathbb{I}_{\mathbb{B}\mathfrak{s}(\mathcal{B})}[\mathcal{B}] = \mathcal{B}.$$

5.2 Theorem on Multi-image for Changeable Sets.

Further $\mathfrak{R}(U)$ will mean the ***range*** of (arbitrary) mapping U.

Definition 8. 1. *The evolution projector* $(\mathbb{T}, \mathcal{X}, U)$ *(where* $\mathbb{T} = (\mathbf{T}, \leq)$*) for base changeable set* $\mathcal{B}$ *will be named* ***bijective*** *if and only if the mapping* U *is bijection from* $\mathbb{B}\mathfrak{s}(\mathcal{B})$ *onto the set* $\mathfrak{R}(U) \subseteq \mathbf{T} \times \mathcal{X}$*.*

2. *Any indexed family* $\mathfrak{P} = ((\mathbb{T}_\alpha, \mathcal{X}_\alpha, U_\alpha) \mid \alpha \in \mathcal{A})$ *(where* $\mathcal{A} \neq \emptyset$*) of bijective evolution projectors for base changeable set we name* ***evolution multi-projector*** *for* $\mathcal{B}$*.*

Theorem 3. *Let,* $\mathfrak{P} = ((\mathbb{T}_\alpha, \mathcal{X}_\alpha, U_\alpha) \mid \alpha \in \mathcal{A})$ *be evolution multi-projector for base changeable set* $\mathcal{B}$*. Then:*

A. *Only one changeable set* $\mathcal{Z}$ *exists, satisfying the following conditions:*

1. $\mathcal{L}k(\mathcal{Z}) = \{(\alpha, U_\alpha[\mathcal{B}, \mathbb{T}_\alpha]) \mid \alpha \in \mathcal{A}\}$*.*

2. *For any areas of perception* $\mathfrak{l} = (\alpha, U_\alpha[\mathcal{B}, \mathbb{T}_\alpha]) \in \mathcal{L}k(\mathcal{Z})$, $\mathfrak{m} = (\beta, U_\beta[\mathcal{B}, \mathbb{T}_\beta]) \in \mathcal{L}k(\mathcal{Z})$ $(\alpha, \beta \in \mathcal{A})$ *and any set* $A \subseteq \mathbb{Bs}(\mathfrak{l}) = U_\alpha(\mathbb{Bs}(\mathcal{B}))$ *the following equality holds:*

$$\langle \mathfrak{m} \leftarrow \mathfrak{l}, \mathcal{Z} \rangle A = U_\beta\left(U_\alpha^{[-1]}(A)\right) = \left\{U_\beta\left(U_\alpha^{[-1]}(\omega)\right) \mid \omega \in A\right\},$$

where $U_\alpha^{[-1]}$ *is the mapping,* ***inverse*** *to* U_α.

B. *Changeable set* $\mathcal{Z}$*, satisfying the conditions 1, 2 is precisely visible.*

Remark 8. Suppose, that a changeable set $\mathcal{Z}$ satisfies condition 1 of Theorem 3. Then for any area of perception $\mathfrak{l} = (\alpha, U_\alpha[\mathcal{B}, \mathbb{T}_\alpha]) \in \mathcal{L}k(\mathcal{Z})$, according to Property 4(1), we have, $\mathtt{ind}(\mathfrak{l}) = \alpha$, $\mathfrak{l}^\wedge = U_\alpha[\mathcal{B}, \mathbb{T}_\alpha]$, and hence, $\mathbb{Bs}(\mathfrak{l}) = \mathbb{Bs}(\mathfrak{l}^\wedge) = \mathbb{Bs}(U_\alpha[\mathcal{B}, \mathbb{T}_\alpha])$. Therefore, by Theorem 2, $\mathbb{Bs}(\mathfrak{l}) = U_\alpha(\mathbb{Bs}(\mathcal{B}))$. Thus, the condition 2 of Theorem 3 is correctly formulated.

Proof of Theorem 3. Let, $\mathfrak{P} = ((\mathbb{T}_\alpha, \mathcal{X}_\alpha, U_\alpha) \mid \alpha \in \mathcal{A})$ be evolution multi-projector for base changeable set $\mathcal{B}$.

A. By Definition 8, for any $\alpha \in \mathcal{A}$ the triple $(\mathbb{T}_\alpha, \mathcal{X}_\alpha, U_\alpha)$, is a bijective evolution projector for $\mathcal{B}$. In accordance with Theorem 2, we put:

$$\mathcal{B}_\alpha := U_\alpha[\mathcal{B}, \mathbb{T}_\alpha] \quad (\alpha \in \mathcal{A}).$$

Since $(\mathbb{T}_\alpha, \mathcal{X}_\alpha, U_\alpha)$ is a bijective evolution projector, then, by the Definition 8, the mapping U_α is one-to-one correspondence. Hence, the inverse mapping $U_\alpha^{[-1]}$ exists (for all $\alpha \in \mathcal{A}$).

For any indexes $\alpha, \beta \in \mathcal{A}$ and any set $A \subseteq \mathbb{Bs}(\mathcal{B}_\alpha)$ we denote:

$$\mathfrak{U}_{\beta,\alpha} A := U_\beta\left(U_\alpha^{[-1]}(A)\right) \tag{4}$$

(note, that, by Theorem 2, $\mathbb{Bs}(\mathcal{B}_\alpha) = U_\alpha(\mathbb{Bs}(\mathcal{B}))$). Hence, $\mathfrak{U}_{\beta,\alpha}$ is the mapping from $2^{\mathbb{Bs}(\mathcal{B}_\alpha)}$ into $2^{\mathbb{Bs}(\mathcal{B}_\beta)} = 2^{U_\beta(\mathbb{Bs}(\mathcal{B}))}$.

It is easy to verify, that the family of mappings $(\mathfrak{U}_{\beta,\alpha} \mid \alpha, \beta \in \mathcal{A})$ possesses the following properties:

1. $\mathfrak{U}_{\alpha,\alpha} A = A$ $(\alpha \in \mathcal{A}, A \subseteq \mathbb{Bs}(\mathcal{B}_\alpha))$;
2. If $\alpha, \beta \in \mathcal{A}$ and $A \subseteq B \subseteq \mathbb{Bs}(\mathcal{B}_\alpha)$, then $\mathfrak{U}_{\beta,\alpha} A \subseteq \mathfrak{U}_{\beta,\alpha} B$.
3. If $\alpha, \beta, \gamma \in \mathcal{A}$ and $A \subseteq \mathbb{Bs}(\mathcal{B}_\alpha)$, then

$$\mathfrak{U}_{\gamma,\beta} \mathfrak{U}_{\beta,\alpha} A = \mathfrak{U}_{\gamma,\alpha} A. \tag{5}$$

Therefore, by [11, Definition 9.6] or [13, Definition 3.1], the triple:

$$\mathcal{Z} = \left(\mathcal{A}, \overleftarrow{\mathcal{B}}, \overleftarrow{\mathfrak{U}}\right), \text{ where } \quad \overleftarrow{\mathcal{B}} = (\mathcal{B}_\alpha \mid \alpha \in \mathcal{A}); \ \overleftarrow{\mathfrak{U}} = (\mathfrak{U}_{\beta,\alpha} \mid \alpha, \beta \in \mathcal{A}).$$

is changeable set. Herewith, according to system of denotations for the changeable sets theory, accepted in [11], [13]:

$$\mathcal{L}k(\mathcal{Z}) = \{(\alpha, \mathcal{B}_\alpha) \mid \alpha \in \mathcal{A}\} = \{(\alpha, U_\alpha[\mathcal{B}, \mathbb{T}_\alpha]) \mid \alpha \in \mathcal{A}\}, \tag{6}$$

and for arbitrary areas of perception $\mathfrak{l} = (\alpha, U_\alpha[\mathcal{B}, \mathbb{T}_\alpha]) \in \mathcal{L}k(\mathcal{Z})$, $\mathfrak{m} = (\beta, U_\beta[\mathcal{B}, \mathbb{T}_\beta]) \in \mathcal{L}k(\mathcal{Z})$ (where $\alpha, \beta \in \mathcal{A}$) and for any set $A \subseteq \mathbb{Bs}(\mathfrak{l}) = \mathbb{Bs}(U_\alpha[\mathcal{B}, \mathbb{T}_\alpha]) = U_\alpha(\mathbb{Bs}(\mathcal{B}))$, by (4) we obtain:

$$\langle \mathfrak{m} \leftarrow \mathfrak{l}, \ \mathcal{Z} \rangle A = \mathfrak{U}_{\beta,\alpha} A = U_\beta\left(U_\alpha^{[-1]}(A)\right). \tag{7}$$

From (6) and (7) it follows, that the changeable set $\mathcal{Z}$ satisfies conditions 1,2 of Theorem 3.

Suppose, that the changeable set $\mathcal{Z}_1$ also satisfies conditions 1,2 of Theorem 3. Then, by the condition 1, $\mathcal{L}k(\mathcal{Z}) = \mathcal{L}k(\mathcal{Z}_1)$. Also, by the condition 2, for arbitrary areas of

perception $\mathfrak{l}, \mathfrak{m} \in \mathcal{L}k(\mathcal{Z}) = \mathcal{L}k(\mathcal{Z}_1)$ it is true the equality: $\langle \mathfrak{m} \leftarrow \mathfrak{l}, \mathcal{Z} \rangle = \langle \mathfrak{m} \leftarrow \mathfrak{l}, \mathcal{Z}_1 \rangle$. Hence, by Assertion 1, we get $\mathcal{Z} = \mathcal{Z}_1$. Thus, changeable set, satisfying the conditions 1,2 of Theorem 3 is unique.

B. Using the equalities (7) and (5), for arbitrary areas of perception $\mathfrak{l}, \mathfrak{m}, \mathfrak{p} \in \mathcal{L}k(\mathcal{Z})$ of kind $\mathfrak{l} = (\alpha, U_\alpha[\mathcal{B}, \mathbb{T}_\alpha])$, $\mathfrak{m} = (\beta, U_\beta[\mathcal{B}, \mathbb{T}_\beta])$, $\mathfrak{p} = (\gamma, U_\gamma[\mathcal{B}, \mathbb{T}_\gamma])$ $(\alpha, \beta, \gamma \in \mathcal{A})$ we get:

$$\langle \mathfrak{p} \leftarrow \mathfrak{m} \rangle \langle \mathfrak{m} \leftarrow \mathfrak{l} \rangle A = \mathfrak{U}_{\gamma,\beta} \mathfrak{U}_{\beta,\alpha} A = \mathfrak{U}_{\gamma,\alpha} A = \langle \mathfrak{p} \leftarrow \mathfrak{l} \rangle A \quad (A \subseteq \mathbb{B}\mathfrak{s}(\mathfrak{l})).$$

Thus, by [12, Theorem 5.1] or [11, Theorem 11.1], the changeable set $\mathcal{Z}$ is precisely visible. $\square$

Definition 9. *Let $\mathfrak{P} = ((\mathbb{T}_\alpha, \mathcal{X}_\alpha, U_\alpha) \mid \alpha \in \mathcal{A})$ be evolution multi-projector for base changeable set $\mathcal{B}$. Changeable set $\mathcal{Z}$, satisfying conditions 1,2 of Theorem 3 will be referred to as* ***evolution multi-image*** *of base changeable set $\mathcal{B}$ relatively the evolution multi-projector $\mathfrak{P}$. This evolution multi-image will be denoted by $\mathcal{Z}\mathsf{im}[\mathfrak{P}, \mathcal{B}]$:*

$$\mathcal{Z}\mathsf{im}[\mathfrak{P}, \mathcal{B}] := \mathcal{Z}.$$

From the theorems 3 and 2, taking into account Property 2(2), Property 3(1) and Remark 3, we immediately deduce the following properties of multi-image for base changeable set.

Properties 6. *Let $\mathfrak{P} = ((\mathbb{T}_\alpha, \mathcal{X}_\alpha, U_\alpha) \mid \alpha \in \mathcal{A})$, where $\mathbb{T}_\alpha = (\mathbf{T}_\alpha, \leq_\alpha)$ $(\alpha \in \mathcal{A})$ be an evolution multi-projector for base changeable set $\mathcal{B}$ and $\mathcal{Z} = \mathcal{Z}\mathsf{im}[\mathfrak{P}, \mathcal{B}]$. Then:*

1. *$\mathcal{L}k(\mathcal{Z}) = \{(\alpha, U_\alpha[\mathcal{B}, \mathbb{T}_\alpha]) \mid \alpha \in \mathcal{A}\}$.*
2. *$\mathcal{I}nd(\mathcal{Z}) = \mathcal{A}$.*
3. *For any area of perception $\mathfrak{l} = (\alpha, U_\alpha[\mathcal{B}, \mathbb{T}_\alpha])$ the following equalities hold:*

$$\begin{aligned} \mathbb{B}\mathfrak{s}(\mathfrak{l}) &= U_\alpha(\mathbb{B}\mathfrak{s}(\mathcal{B})) = \{U_\alpha(\omega) \mid \omega \in \mathbb{B}\mathfrak{s}(\mathcal{B})\}; \\ \mathfrak{B}\mathfrak{s}(\mathfrak{l}) &= \{\mathsf{bs}(U_\alpha(\omega)) \mid \omega \in \mathbb{B}\mathfrak{s}(\mathcal{B})\}; \\ \mathbb{T}\mathbf{m}(\mathfrak{l}) &= \mathbb{T}_\alpha; \quad \mathbf{Tm}(\mathfrak{l}) = \mathbf{T}_\alpha. \end{aligned}$$

4. *Let, $\mathfrak{l} = (\alpha, U_\alpha[\mathcal{B}, \mathbb{T}_\alpha]) \in \mathcal{L}k(\mathcal{Z})$, where $\alpha \in \mathcal{A}$. Suppose, that $\widetilde{\omega}_1, \widetilde{\omega}_2 \in \mathbb{B}\mathfrak{s}(\mathfrak{l})$ and $\mathsf{tm}(\widetilde{\omega}_1) \neq \mathsf{tm}(\widetilde{\omega}_2)$. Then $\widetilde{\omega}_1$ and $\widetilde{\omega}_2$ are united by fate in $\mathfrak{l}$ if and only if there exist united by fate in $\mathcal{B}$ elementary-time states $\omega_1, \omega_2 \in \mathbb{B}\mathfrak{s}(\mathcal{B})$ such, that $\widetilde{\omega}_1 = U_\alpha(\omega_1)$, $\widetilde{\omega}_2 = U_\alpha(\omega_2)$.*

Let $\mathcal{B}$ be any base changeable set, and X be any set, containing $\mathfrak{B}\mathfrak{s}(\mathcal{B})$ $(\mathfrak{B}\mathfrak{s}(\mathcal{B}) \subseteq X)$. Recall ([13, Example 3.2], [11, Example 10.2]), that any set $\mathbb{U}$ of bijections, defined on $\mathbf{Tm}(\mathcal{B}) \times X$:

$$U : \mathbf{Tm}(\mathcal{B}) \times X \longleftrightarrow \mathbf{Tm}(\mathcal{B}) \times X \quad (U \in \mathbb{U})$$

is named ***transforming set of bijections*** relatively the base changeable set $\mathcal{B}$ on X.

Let $\mathbb{U}$ be transforming set of bijections relatively $\mathcal{B}$ on X. Then, by the Definition 6, any mapping $U \in \mathbb{U}$ generates the evolution projector, $\left(\mathbb{T}\mathbf{m}(\mathcal{B}), X, U_{\restriction \mathbb{B}\mathfrak{s}(\mathcal{B})}\right)$, where $U_{\restriction \mathbb{B}\mathfrak{s}(\mathcal{B})}$ is the restriction of the mapping U onto the set $\mathbb{B}\mathfrak{s}(\mathcal{B}) \subseteq \mathbf{Tm}(\mathcal{B}) \times X$ (henceforth, where it does not cause confusion, we identify the mapping $U_{\restriction \mathbb{B}\mathfrak{s}(\mathcal{B})}$ with the mapping U; under this identifying, we can consider, that $\left(\mathbb{T}\mathbf{m}(\mathcal{B}), X, U_{\restriction \mathbb{B}\mathfrak{s}(\mathcal{B})}\right) = (\mathbb{T}\mathbf{m}(\mathcal{B}), X, U))$. Hence, the indexed family:

$$\mathfrak{P}_{\mathcal{B}}[\mathbb{U}] = ((\mathbb{T}\mathbf{m}(\mathcal{B}), X, U) \mid U \in \mathbb{U})$$

is evolution multi-projector for $\mathcal{B}$. In this particular case we have the changeable set:

$$\mathcal{Z}\mathsf{im}(\mathbb{U}, \mathcal{B}) = \mathcal{Z}\mathsf{im}[\mathfrak{P}_{\mathcal{B}}[\mathbb{U}], \mathcal{B}],$$

that is multi-figurative image of base changeable set $\mathcal{B}$ relatively the transforming set of bijections $\mathbb{U}$ (in the sense of [11, 13]).

From the item B of Theorem 3 it follows, that the changeable sets of kind $\mathcal{Z}\mathsf{im}[\mathfrak{P}, \mathcal{B}]$ and $\mathcal{Z}\mathsf{im}(\mathbb{U}, \mathcal{B})$ are precisely visible. Therefore, we deliver the following corollary of Theorem 3:

Corollary 1. *If* $\mathcal{Z} = \mathcal{Z}\mathsf{im}\left[\mathfrak{P},\mathcal{B}\right]$, *where* $\mathfrak{P} = ((\mathbb{T}_\alpha, \mathcal{X}_\alpha, U_\alpha) \mid \alpha \in \mathcal{A})$, *then for any areas of perception* $\mathfrak{l} = (\alpha, U_\alpha[\mathcal{B}, \mathbb{T}_\alpha]) \in \mathcal{L}k(\mathcal{Z})$, $\mathfrak{m} = (\beta, U_\beta[\mathcal{B}, \mathbb{T}_\beta]) \in \mathcal{L}k(\mathcal{Z})$ $(\alpha, \beta \in \mathcal{A})$ *the following equality is performed:*

$$\langle !\, \mathfrak{m} \leftarrow \mathfrak{l},\ \mathcal{Z}\rangle\, \omega = U_\beta\left(U_\alpha^{[-1]}(\omega)\right) \quad (\omega \in \mathbb{B}\mathfrak{s}(\mathfrak{l}) = U_\alpha(\mathbb{B}\mathfrak{s}(\mathcal{B}))).$$

5.3 Theorem on Multi-image for Kinematic Sets.

Definition 10.

1. *The ordered composition of five sets* $(\mathbb{T}, \mathcal{X}, U, \mathfrak{Q}, k)$ *will be named as bijective* ***kinematic projector*** *for base changeable set* $\mathcal{B}$ *if and only if:*

   1.1. $(\mathbb{T}, \mathcal{X}, U)$ *is bijective evolution projector for* $\mathcal{B}$.

   1.2. $\mathfrak{Q}$ *is a coordinate space.*

   1.3. k *is a mapping from* $\mathcal{X}$ *into* $\mathbf{Zk}(\mathfrak{Q})$.

2. *Any indexed family* $\mathfrak{P} = ((\mathbb{T}_\alpha, \mathcal{X}_\alpha, U_\alpha, \mathfrak{Q}_\alpha, k_\alpha) \mid \alpha \in \mathcal{A})$ *(where* $\mathcal{A} \neq \emptyset$*) of bijective kinematic projectors for base changeable set* $\mathcal{B}$ *we name* ***kinematic multi-projector*** *for* $\mathcal{B}$.

Remark 9. Henceforward we will consider only bijective kinematic projectors. That is why we will use the term "kinematic projector" instead of the term "bijective kinematic projector".

Let $\mathfrak{P} = ((\mathbb{T}_\alpha, \mathcal{X}_\alpha, U_\alpha, \mathfrak{Q}_\alpha, k_\alpha) \mid \alpha \in \mathcal{A})$ be any kinematic multi-projector for $\mathcal{B}$. Denote:

$$\mathfrak{P}^{[e]} := ((\mathbb{T}_\alpha, \mathcal{X}_\alpha, U_\alpha) \mid \alpha \in \mathcal{A}).$$

By the definitions 10 and 8, $\mathfrak{P}^{[e]}$ is evolution multi-projector for $\mathcal{B}$.

Theorem 4. *Let* $\mathfrak{P} = ((\mathbb{T}_\alpha, \mathcal{X}_\alpha, U_\alpha, \mathfrak{Q}_\alpha, k_\alpha) \mid \alpha \in \mathcal{A})$ *be a kinematic multi-projector for a base changeable set* $\mathcal{B}$. *Then:*
A) *Only one kinematic set* $\mathfrak{C}$ *exists, satisfying the following conditions:*

1. $\mathbb{BE}(\mathfrak{C}) = \mathcal{Z}\mathsf{im}\left[\mathfrak{P}^{[e]}, \mathcal{B}\right]$.
2. *For any reference frame* $\mathfrak{l} = (\alpha, U_\alpha[\mathcal{B}, \mathbb{T}_\alpha]) \in \mathcal{L}k(\mathfrak{C})$ *(where* $\alpha \in \mathcal{A}$*) the following equalities are performed:*

$$\textbf{2.1)}\ \mathsf{BG}(\mathfrak{l}) = \mathfrak{Q}_\alpha; \qquad \textbf{2.2)}\ \mathfrak{q}_\mathfrak{l}(x) = k_\alpha(x) \quad (x \in \mathfrak{Bs}(\mathfrak{l})).$$

B) *Kinematic set* $\mathfrak{C}$, *satisfying the conditions 1,2 is precisely visible.*

Proof. Let $\mathfrak{P} = ((\mathbb{T}_\alpha, \mathcal{X}_\alpha, U_\alpha, \mathfrak{Q}_\alpha, k_\alpha) \mid \alpha \in \mathcal{A})$ (where $\mathbb{T}_\alpha = (\mathbf{T}_\alpha, \leq_\alpha)$, $\alpha \in \mathcal{A}$) be a kinematic multi-projector for $\mathcal{B}$.

A) Put:

$$\mathcal{Z} := \mathcal{Z}\mathsf{im}\left[\mathfrak{P}^{[e]}, \mathcal{B}\right].$$

Then, according to Theorem 3:

$$\mathcal{L}k(\mathcal{Z}) = \{(\alpha, U_\alpha[\mathcal{B}, \mathbb{T}_\alpha]) \mid \ \alpha \in \mathcal{A}\}.$$

Consider any fixed area of perception $\mathfrak{l} = (\alpha, U_\alpha[\mathcal{B}, \mathbb{T}_\alpha]) \in \mathcal{L}k(\mathcal{Z})$ (where $\alpha \in \mathcal{A}$). Denote:

$$\mathfrak{Q}^{(\mathfrak{l})} := \mathfrak{Q}_\alpha.$$

The ordered five-elements composition $(\mathbb{T}_\alpha, \mathcal{X}_\alpha, U_\alpha, \mathfrak{Q}_\alpha, k_\alpha)$ is a kinematic projector. Hence, by the Definition 10, the triple $(\mathbb{T}_\alpha, \mathcal{X}_\alpha, U_\alpha) = ((\mathbf{T}_\alpha, \leq_\alpha), \mathcal{X}_\alpha, U_\alpha)$ is evolution projector for $\mathcal{B}$. Consequently, by the definition of evolution projector (Definition 6), U_α is the mapping of kind $U_\alpha : \mathbb{B}\mathfrak{s}(\mathcal{B}) \mapsto \mathbf{T}_\alpha \times \mathcal{X}_\alpha$. Therefore, by Property 6(3), we obtain:

$$\mathfrak{Bs}(\mathfrak{l}) = \{\mathsf{bs}(U_\alpha(\omega)) \mid \omega \in \mathbb{B}\mathfrak{s}(\mathcal{B})\} \subseteq \mathcal{X}_\alpha.$$

For an arbitrary $x \in \mathfrak{Bs}(\mathfrak{l})$ we denote:

$$k^{(\mathfrak{l})}(x) := k_\alpha(x).$$

According to the definition of a kinematic projector (Definition 10) k_α is the mapping from $\mathcal{X}_\alpha$ into $\mathbf{Zk}\left(\mathfrak{Q}_\alpha\right) = \mathbf{Zk}\left(\mathfrak{Q}^{(\mathfrak{l})}\right)$. Hence, $k^{(\mathfrak{l})}$ is the mapping from $\mathfrak{Bs}(\mathfrak{l})$ into $\mathbf{Zk}\left(\mathfrak{Q}^{(\mathfrak{l})}\right)$.

Hence, by the Definition 4 (item 2), the pair

$$\mathfrak{C} = \left(\mathcal{Z},\ \left(\left(\mathfrak{Q}^{(\mathfrak{l})},\ k^{(\mathfrak{l})}\right) \mid \mathfrak{l} \in \mathcal{L}k\left(\mathcal{Z}\right)\right)\right) \tag{8}$$

is a kinematic set. Herewith, taking into account the system of denotations, accepted in the subsection 3.2.2, we get:

$$\mathbb{BE}(\mathfrak{C}) = \mathcal{Z} = \mathcal{Z}\mathsf{im}\left[\mathfrak{P}^{[e]}\right],$$

and for any reference frame $\mathfrak{l} = (\alpha, U_\alpha\left[\mathcal{B}, \mathbb{T}_\alpha\right]) \in \mathcal{L}k\left(\mathfrak{C}\right)$, where $\alpha \in \mathcal{A}$ we have:

$$\begin{aligned} &\mathsf{BG}(\mathfrak{l}) = \mathfrak{Q}^{(\mathfrak{l})} = \mathfrak{Q}_\alpha; \\ &\mathfrak{q}_\mathfrak{l}(x) = k^{(\mathfrak{l})}(x) = k_\alpha(x) \quad (x \in \mathfrak{Bs}(\mathfrak{l})). \end{aligned}$$

Thus, the kinematic set $\mathfrak{C}$ satisfies the conditions 1,2 from the item **A)** of Theorem 4.

Now, we are going to prove, that kinematic set $\mathfrak{C}$, satisfying the conditions 1,2 from the item **A)** of Theorem 4 is unique. Assume, that the kinematic set $\mathfrak{C}_1$ also satisfies the conditions 1,2 from the item **A)** of Theorem 4. Then, by the condition 1, $\mathbb{BE}\left(\mathfrak{C}\right) = \mathcal{Z} = \mathbb{BE}\left(\mathfrak{C}_1\right)$. Hence,

$$\mathcal{L}k\left(\mathfrak{C}\right) = \mathcal{L}k\left(\mathcal{Z}\right) = \mathcal{L}k\left(\mathfrak{C}_1\right),$$

moreover, for any reference frames $\mathfrak{l}, \mathfrak{m} \in \mathcal{L}k\left(\mathfrak{C}\right) = \mathcal{L}k\left(\mathfrak{C}_1\right)$ we have:

$$\langle \mathfrak{m} \leftarrow \mathfrak{l}, \mathfrak{C} \rangle = \langle \mathfrak{m} \leftarrow \mathfrak{l}, \mathcal{Z} \rangle = \langle \mathfrak{m} \leftarrow \mathfrak{l}, \mathfrak{C}_1 \rangle.$$

Further, by the condition 2, for any reference frame $\mathfrak{l} = (\alpha, U_\alpha\left[\mathcal{B}, \mathbb{T}_\alpha\right]) \in \mathcal{L}k\left(\mathfrak{C}\right) = \mathcal{L}k\left(\mathfrak{C}_1\right)$ we deliver:

$$\mathsf{BG}\left(\mathfrak{l};\ \mathfrak{C}\right) = \mathfrak{Q}_\alpha = \mathsf{BG}\left(\mathfrak{l};\ \mathfrak{C}_1\right); \qquad \mathfrak{q}_\mathfrak{l}\left(x,\ \mathfrak{C}\right) = k_\alpha(x) = \mathfrak{q}_\mathfrak{l}\left(x,\ \mathfrak{C}_1\right) \quad \left(x \in \mathfrak{Bs}(\mathfrak{l})\right).$$

Therefore, by Assertion 4, $\mathfrak{C} = \mathfrak{C}_1$.

B) Since the changeable set $\mathcal{Z}$ is precisely visible, then the kinematic set $\mathfrak{C}$, represented, by the formula (8), according to the item d) of the subsection 3.2.2, also is precisely visible. $\square$

Definition 11. *Let,* $\mathfrak{P} = \left((\mathbb{T}_\alpha, \mathcal{X}_\alpha, U_\alpha, \mathfrak{Q}_\alpha, k_\alpha) \mid \alpha \in \mathcal{A}\right)$ *be a kinematic multi-projector for a base changeable set* $\mathcal{B}$. *The kinematic set* $\mathfrak{C}$, *satisfying the conditions 1,2 of Theorem 4 will be named as* ***kinematic multi-image*** *of base changeable set* $\mathcal{B}$ *relatively the kinematic multi-projector* $\mathfrak{P}$. *This kinematic set will be denoted via* $\mathfrak{Kim}\left[\mathfrak{P}, \mathcal{B}\right]$:

$$\mathfrak{Kim}\left[\mathfrak{P}, \mathcal{B}\right] := \mathfrak{C}.$$

Applying Properties 6, Corollary 1 and Theorem 4, we obtain the following properties for kinematic multi-image of base changeable set.

Properties 7. *Let,* $\mathfrak{P} = \left((\mathbb{T}_\alpha, \mathcal{X}_\alpha, U_\alpha, \mathfrak{Q}_\alpha, k_\alpha) \mid \alpha \in \mathcal{A}\right)$ *be a kinematic multi-projector for* $\mathcal{B}$ *(where* $\mathbb{T}_\alpha = (\mathbf{T}_\alpha, \leq_\alpha)$, $\alpha \in \mathcal{A}$*). Suppose, that* $\mathfrak{C} = \mathfrak{Kim}\left[\mathfrak{P}, \mathcal{B}\right]$. *Then:*

1. $\mathcal{L}k\left(\mathfrak{C}\right) = \left\{(\alpha, U_\alpha\left[\mathcal{B}, \mathbb{T}_\alpha\right]) \mid \alpha \in \mathcal{A}\right\}$.
2. $\mathcal{I}nd\left(\mathfrak{C}\right) = \mathcal{A}$.
3. *For any reference frame* $\mathfrak{l} = (\alpha, U_\alpha\left[\mathcal{B}, \mathbb{T}_\alpha\right])$ *the following equalities hold:*

$$\mathbb{Bs}(\mathfrak{l}) = U_\alpha\left(\mathbb{Bs}(\mathcal{B})\right) = \left\{U_\alpha(\omega) \mid \omega \in \mathbb{Bs}(\mathcal{B})\right\};$$

$$\mathfrak{Bs}(\mathfrak{l}) = \{\mathsf{bs}\,(U_\alpha(\omega)) \mid \omega \in \mathbb{Bs}(\mathcal{B})\}\,;$$
$$\mathbb{T}\mathbf{m}(\mathfrak{l}) = \mathbb{T}_\alpha; \quad \mathbf{Tm}(\mathfrak{l}) = \mathbf{T}_\alpha;$$
$$\mathbf{Zk}(\mathfrak{l}) = \mathbf{Zk}\,(\mathsf{BG}(\mathfrak{l})) = \mathbf{Zk}\,(\mathfrak{Q}_\alpha)\,;$$
$$\mathbb{M}k(\mathfrak{l}) = \mathbf{Tm}(\mathfrak{l}) \times \mathbf{Zk}(\mathfrak{l}) = \mathbf{T}_\alpha \times \mathbf{Zk}\,(\mathfrak{Q}_\alpha)\,;$$
$$\mathfrak{q}_{\mathfrak{l}}(x) = k_\alpha(x) \quad (x \in \mathfrak{Bs}(\mathfrak{l}));$$
$$\mathbf{Q}^{\langle \mathfrak{l}\rangle}(\omega) = (\mathsf{tm}\,(\omega)\,, \mathfrak{q}_{\mathfrak{l}}(\mathsf{bs}\,(\omega))) = (\mathsf{tm}\,(\omega)\,, k_\alpha(\mathsf{bs}\,(\omega))) \qquad (\omega \in \mathbb{Bs}(\mathfrak{l}))\,.$$

4. *Let, $\mathfrak{l} = (\alpha, U_\alpha[\mathcal{B}, \mathbb{T}_\alpha]) \in \mathcal{L}k\,(\mathfrak{C})$, where $\alpha \in \mathcal{A}$. Suppose, that $\widetilde{\omega}_1, \widetilde{\omega}_2 \in \mathbb{Bs}(\mathfrak{l})$ and $\mathsf{tm}\,(\widetilde{\omega}_1) \neq \mathsf{tm}\,(\widetilde{\omega}_2)$. Then $\widetilde{\omega}_1$ and $\widetilde{\omega}_2$ are united by fate in $\mathfrak{l}$ if and only if there exist united by fate in $\mathcal{B}$ elementary-time states $\omega_1, \omega_2 \in \mathbb{Bs}(\mathcal{B})$ such, that $\widetilde{\omega}_1 = U_\alpha(\omega_1)$, $\widetilde{\omega}_2 = U_\alpha(\omega_2)$.*
5. *For any reference frames $\mathfrak{l} = (\alpha, U_\alpha[\mathcal{B}, \mathbb{T}_\alpha]) \in \mathcal{L}k\,(\mathfrak{C})$, $\mathfrak{m} = (\beta, U_\beta[\mathcal{B}, \mathbb{T}_\beta]) \in \mathcal{L}k\,(\mathfrak{C})$ ($\alpha, \beta \in \mathcal{A}$) the following equality holds:*

$$\langle !\, \mathfrak{m} \leftarrow \mathfrak{l},\, \mathfrak{C}\rangle\, \omega = U_\beta\left(U_\alpha^{[-1]}(\omega)\right) \quad (\omega \in \mathbb{Bs}(\mathfrak{l}) = U_\alpha\,(\mathbb{Bs}(\mathcal{B})))\,.$$

6 Kinematic Sets, Generated by Special Relativity and its Tachyon Extensions.

Let, $\mathfrak{Q}$ be a coordinate space, $\mathcal{B}$ be a base changeable set such, that $\mathfrak{Bs}(\mathcal{B}) \subseteq \mathbf{Zk}(\mathfrak{Q})$ (such base changeable set $\mathcal{B}$ exists, because, for example, we may put $\mathcal{B} := \mathcal{A}t\,(\mathbb{T}, \mathcal{R})$, where $\mathcal{R}$ is a system of abstract trajectories from the linear ordered set $\mathbb{T}$ to a set $\mathbf{M} \subseteq \mathbf{Zk}(\mathfrak{Q})$, where the definition of $\mathcal{A}t\,(\mathbb{T}, \mathcal{R})$ can be found in [11, 14]). Let $\mathbb{U}$ be any transforming set of bijections relatively the $\mathcal{B}$ on $\mathbf{Zk}(\mathfrak{Q})$. Then, any mapping $\mathbf{U} \in \mathbb{U}$ is the mapping of kind, $\mathbf{U} : \mathbf{Tm}(\mathcal{B}) \times \mathbf{Zk}(\mathfrak{Q}) \longleftrightarrow \mathbf{Tm}(\mathcal{B}) \times \mathbf{Zk}(\mathfrak{Q})$, where $\mathbb{Bs}(\mathcal{B}) \subseteq \mathbf{Tm}(\mathcal{B}) \times \mathfrak{Bs}(\mathcal{B}) \subseteq \mathbf{Tm}(\mathcal{B}) \times \mathbf{Zk}(\mathfrak{Q})$. Hence, the set of bijections $\mathbb{U}$ generates the kinematic multi-projector $\widehat{\mathbb{U}} := \left(\left(\mathbb{T}\mathbf{m}(\mathcal{B}), \mathbf{Zk}(\mathfrak{Q}), \mathbf{U}, \mathfrak{Q}, \mathbb{I}_{\mathbf{Zk}(\mathfrak{Q})}\right) \mid \mathbf{U} \in \mathbb{U}\right)$ for $\mathcal{B}$, where $\mathbb{I}_{\mathbf{Zk}(\mathfrak{Q})}$ is the identity mapping on $\mathbf{Zk}(\mathfrak{Q})$. Denote:

$$\mathfrak{Kim}\,(\mathbb{U}, \mathcal{B}, \mathfrak{Q}) := \mathfrak{Kim}\left[\widehat{\mathbb{U}}, \mathcal{B}\right].$$

Theorem 5. *The kinematic set $\mathfrak{C} = \mathfrak{Kim}\,(\mathbb{U}, \mathcal{B}, \mathfrak{Q})$ allows universal coordinate transform. Moreover, $\mathcal{L}k\,(\mathfrak{C}) = ((\mathbf{U}, \mathbf{U}\,[\mathcal{B}]) \mid \mathbf{U} \in \mathbb{U})$, and the system of mappings $\left(\widetilde{Q}_{\mathfrak{m},\mathfrak{l}}\right)_{\mathfrak{l},\mathfrak{m} \in \mathcal{L}k(\mathfrak{C})}$:*

$$\widetilde{Q}_{\mathfrak{m},\mathfrak{l}}(\mathrm{w}) = \mathbf{V}\left(\mathbf{U}^{[-1]}(\mathrm{w})\right), \quad \mathrm{w} \in \mathbb{M}k(\mathfrak{l}) = \mathbf{Tm}(\mathcal{B}) \times \mathbf{Zk}(\mathfrak{Q}) \qquad (9)$$
$$(\,\mathfrak{l} = (\mathbf{U}, \mathbf{U}\,[\mathcal{B}]) \in \mathcal{L}k\,(\mathfrak{C})\,, \quad \mathfrak{m} = (\mathbf{V}, \mathbf{V}\,[\mathcal{B}]) \in \mathcal{L}k\,(\mathfrak{C})\,)$$

is universal coordinate transform for $\mathfrak{C}$.

Proof. Let, $\mathfrak{Q}$ be a coordinate space and $\mathbb{U}$ be transforming set of bijections relatively the base changeable set $\mathcal{B}$ ($\mathfrak{Bs}(\mathcal{B}) \subseteq \mathbf{Zk}(\mathfrak{Q})$) on $\mathbf{Zk}(\mathfrak{Q})$. Denote $\mathfrak{C} = \mathfrak{Kim}\,(\mathbb{U}, \mathcal{B}, \mathfrak{Q})$. Then, $\mathfrak{C} = \mathfrak{Kim}\left[\widehat{\mathbb{U}}, \mathcal{B}\right]$, where $\widehat{\mathbb{U}} = \left(\left(\mathbb{T}\mathbf{m}(\mathcal{B}), \mathbf{Zk}(\mathfrak{Q}), \mathbf{U}, \mathfrak{Q}, \mathbb{I}_{\mathbf{Zk}(\mathfrak{Q})}\right) \mid \mathbf{U} \in \mathbb{U}\right)$. Hence, according to Property 7(1), $\mathcal{L}k\,(\mathfrak{C}) = \{(\mathbf{U}, \mathbf{U}\,[\mathcal{B}]) \mid \mathbf{U} \in \mathbb{U}\}$. And, by Property 7(3), for an arbitrary reference frame $\mathfrak{l} = (\mathbf{U}, \mathbf{U}\,[\mathcal{B}]) \in \mathcal{L}k\,(\mathfrak{C})$ we have: $\mathfrak{Bs}(\mathfrak{l}) = \{\mathsf{bs}\,(\mathbf{U}(\omega)) \mid \omega \in \mathbb{Bs}(\mathcal{B})\} \subseteq \mathbf{Zk}(\mathfrak{Q})$. Herewith, by Theorem 4, $\mathfrak{q}_{\mathfrak{l}}\,(x, \mathfrak{C}) = x \quad (\forall\, x \in \mathfrak{Bs}(\mathfrak{l}))$. Hence:

$$\mathbf{Q}^{\langle \mathfrak{l}\rangle}(\omega; \mathfrak{C}) = (\mathsf{tm}\,(\omega)\,, \mathfrak{q}_{\mathfrak{l}}(\mathsf{bs}\,(\omega))) = (\mathsf{tm}\,(\omega)\,, \mathsf{bs}\,(\omega)) = \omega \quad (\mathfrak{l} \in \mathcal{L}k\,(\mathfrak{C})\,,\ \omega \in \mathbb{Bs}(\mathfrak{l}))\,.$$

Using the last equality and Property 7(5), for arbitrary reference frames $\mathfrak{l} = (\mathbf{U}, \mathbf{U}\,[\mathcal{B}]) \in \mathcal{L}k\,(\mathfrak{C})$, $\mathfrak{m} = (\mathbf{V}, \mathbf{V}\,[\mathcal{B}]) \in \mathcal{L}k\,(\mathfrak{C})$ $(\mathbf{U}, \mathbf{V} \in \mathbb{U})$ we obtain:

$$\mathbf{Q}^{\langle \mathfrak{m} \leftarrow \mathfrak{l}\rangle}(\omega; \mathfrak{C}) = \mathbf{Q}^{\langle \mathfrak{m}\rangle}(\langle !\, \mathfrak{m} \leftarrow \mathfrak{l}\rangle\, \omega) = \langle !\, \mathfrak{m} \leftarrow \mathfrak{l}\rangle\, \omega =$$

$$= \mathbf{V}\left(\mathbf{U}^{[-1]}(\omega)\right) = \mathbf{V}\left(\mathbf{U}^{[-1]}\left(\mathbf{Q}^{\langle \mathfrak{l}\rangle}(\omega)\right)\right) = \widetilde{Q}_{\mathfrak{m},\mathfrak{l}}\left(\mathbf{Q}^{\langle \mathfrak{l}\rangle}(\omega)\right).$$

It is not hard to verify, that the system of mappings $\left(\widetilde{Q}_{\mathfrak{m},\mathfrak{l}}\right)_{\mathfrak{l},\mathfrak{m}\in\mathcal{L}k(\mathfrak{C})}$ satisfies the conditions (2). Therefore, by the Definition 5 (item 4), we see, that $\left(\widetilde{Q}_{\mathfrak{m},\mathfrak{l}}\right)_{\mathfrak{l},\mathfrak{m}\in\mathcal{L}k(\mathfrak{C})}$ is universal coordinate transform for $\mathfrak{C}$. $\square$

Let $(\mathfrak{H}, \|\cdot\|, \langle\cdot,\cdot\rangle)$ be a Hilbert space over the real field and $\mathcal{L}(\mathfrak{H})$ be the space of (homogeneous) linear continuous operators over the space $\mathfrak{H}$. Denote by $\mathcal{L}^{\times}(\mathfrak{H})$ the space of all linear continuous operators over the space $\mathfrak{H}$, including non-homogeneous operators, that is $\mathcal{L}^{\times}(\mathfrak{H}) = \left\{\mathbf{A}_{[\mathbf{a}]} \mid \mathbf{A}\in\mathcal{L}(\mathfrak{H}),\ \mathbf{a}\in\mathfrak{H}\right\}$, where $\mathbf{A}_{[\mathbf{a}]}x = \mathbf{A}x + \mathbf{a}$, $x\in\mathfrak{H}$. The Hilbert space $\mathfrak{H}$ generates the coordinate space $\widehat{\mathfrak{H}} = (\mathfrak{H}, \mathcal{T}_{\mathfrak{H}}, \mathbb{L}_{\mathfrak{H}}, \rho_{\mathfrak{H}}, \|\cdot\|, \langle\cdot,\cdot\rangle)$, where $\rho_{\mathfrak{H}}$ and $\mathcal{T}_{\mathfrak{H}}$ are metrics and topology, generated by the norm $\|\cdot\|$ on the space $\mathfrak{H}$, as well as $\mathbb{L}_{\mathfrak{H}}$ is the natural linear structure of the space $\mathfrak{H}$. The ***Minkowski space*** over the Hilbert space $\mathfrak{H}$ is defined as the Hilbert space $\mathcal{M}(\mathfrak{H}) = \mathbb{R}\times\mathfrak{H} = \{(t,x) \mid t\in\mathbb{R},\ x\in\mathfrak{H}\}$, equipped by the inner product and norm: $\langle \mathrm{w}_1, \mathrm{w}_2\rangle = \langle \mathrm{w}_1, \mathrm{w}_2\rangle_{\mathcal{M}(\mathfrak{H})} = t_1t_2 + \langle x_1, x_2\rangle$, $\|\mathrm{w}_1\| = \|\mathrm{w}_1\|_{\mathcal{M}(\mathfrak{H})} = \left(t_1^2 + \|x_1\|^2\right)^{1/2}$ ($\mathrm{w}_i = (t_i, x_i)\in\mathcal{M}(\mathfrak{H})$, $i\in\{1,2\}$) ([8,9]). In the space $\mathcal{M}(\mathfrak{H})$ we select the next subspaces:

$$\mathfrak{H}_0 := \{(t,\mathbf{0}) \mid t\in\mathbb{R}\}, \quad \mathfrak{H}_1 := \{(0,x) \mid x\in\mathfrak{H}\},$$

with $\mathbf{0}$ being zero vector. Then, $\mathcal{M}(\mathfrak{H}) = \mathfrak{H}_0 \oplus \mathfrak{H}_1$, where $\oplus$ means the orthogonal sum of subspaces. Denote: $\mathbf{e}_0 := (1,\mathbf{0})\in\mathcal{M}(\mathfrak{H})$. Now, we introduce the orthogonal projectors on the subspaces $\mathfrak{H}_0$ and $\mathfrak{H}_1$:

$$\begin{aligned} &\mathbf{X}\mathrm{w} = (0,x)\in\mathfrak{H}_1; \quad \widehat{\mathbf{T}}\mathrm{w} = (t,\mathbf{0}) = \mathcal{T}(\mathrm{w})\,\mathbf{e}_0 \in \mathfrak{H}_0, \\ &\qquad\qquad\qquad \text{where } \mathcal{T}(\mathrm{w}) = t \quad (\mathrm{w} = (t,x)\in\mathcal{M}(\mathfrak{H})). \end{aligned} \tag{10}$$

Any vector $V\in\mathfrak{H}_1$ generates the following subspaces in the space $\mathfrak{H}_1$.

$$\mathfrak{H}_1[V] = \mathbf{span}\,\{V\}; \quad \mathfrak{H}_{1\perp}[V] = \mathfrak{H}_1 \ominus \mathfrak{H}_1[V] = \{x\in\mathfrak{H}_1 \mid \langle x, V\rangle = 0\},$$

where $\mathbf{span}\,M$ denotes the linear span of the set $M\subseteq\mathcal{M}(\mathfrak{H})$. We will denote by $\mathbf{X}_1[V]$ and $\mathbf{X}_1^{\perp}[V]$ the orthogonal projectors onto the subspaces $\mathfrak{H}_1[V]$ and $\mathfrak{H}_{1\perp}[V]$:

$$\mathbf{X}_1[V]\,\mathrm{w} = \begin{cases} \langle V, \mathrm{w}\rangle \|V\|^{-2} V, & V\neq\mathbf{0} \\ \mathbf{0}, & V = \mathbf{0} \end{cases}, \ \mathrm{w}\in\mathcal{M}(\mathfrak{H}); \quad \mathbf{X}_1^{\perp}[V] = \mathbf{X} - \mathbf{X}_1[V]. \tag{11}$$

Then for any vector $V\in\mathfrak{H}_1$ we obtain the equality:

$$\widehat{\mathbf{T}} + \mathbf{X} = \widehat{\mathbf{T}} + \mathbf{X}_1[V] + \mathbf{X}_1^{\perp}[V] = \mathbb{I}_{\mathcal{M}(\mathfrak{H})}, \tag{12}$$

where $\mathbb{I}_{\mathcal{M}(\mathfrak{H})}$ is the identity operator on $\mathcal{M}(\mathfrak{H})$.

Denote via $\mathbf{Pk}(\mathfrak{H})$ the set of all operators $\mathbf{S}\in\mathcal{L}^{\times}(\mathcal{M}(\mathfrak{H}))$, which has the continuous inverse operator $\mathbf{S}^{-1}\in\mathcal{L}^{\times}(\mathcal{M}(\mathfrak{H}))$. Operators $\mathbf{S}\in\mathbf{Pk}(\mathfrak{H})$ will be named as ***coordinate transform operators***. Let, $\mathcal{B}$ be any base changeable set such, that $\mathfrak{Bs}(\mathcal{B})\subseteq\mathfrak{H} = \mathbf{Zk}\left(\widehat{\mathfrak{H}}\right)$ and $\mathbb{T}\mathbf{m}(\mathcal{B}) = (\mathbb{R},\leq)$, where $\leq$ is the standard order in the field of real numbers $\mathbb{R}$. Then $\mathbb{B}\mathfrak{s}(\mathcal{B})\subseteq\mathbb{R}\times\mathfrak{H} = \mathcal{M}(\mathfrak{H})$. Any set $\mathbb{S}\subseteq\mathbf{Pk}(\mathfrak{H})$ is transforming set of bijections relatively the $\mathcal{B}$ on $\mathfrak{H} = \mathbf{Zk}\left(\widehat{\mathfrak{H}}\right)$. Therefore, we can put:

$$\mathfrak{Kim}(\mathbb{S},\mathcal{B};\,\mathfrak{H}) := \mathfrak{Kim}\left(\mathbb{S},\mathcal{B},\widehat{\mathfrak{H}}\right).$$

Now, we deduce the following corollary from Theorem 5.

Corollary 2. *The kinematic set* $\mathfrak{Kim}(\mathbb{S},\mathcal{B};\,\mathfrak{H})$ *allows universal coordinate transform.*

Recall, that an operator $U \in \mathcal{L}(\mathfrak{H})$ is referred to as ***unitary*** on $\mathfrak{H}$ if and only if $\exists\, U^{-1} \in \mathcal{L}(\mathfrak{H})$ and $\forall\, x \in \mathfrak{H}$ $\|Ux\| = \|x\|$. Denote:

$$\mathfrak{U}(\mathfrak{H}_1) = \{U \in \mathcal{L}(\mathfrak{H}_1) \mid U \text{ is unitary on } \mathfrak{H}_1\}; \quad \mathbf{B}_1(\mathfrak{H}_1) = \{x \in \mathfrak{H}_1 \mid \|x\| = 1\}.$$

Consider any fixed values $c \in (0, \infty]$, $\lambda \in [0, \infty] \setminus \{c\}$, $s \in \{-1, 1\}$, $J \in \mathfrak{U}(\mathfrak{H}_1)$, $\mathbf{n} \in \mathbf{B}_1(\mathfrak{H}_1)$, and $\mathbf{a} \in \mathcal{M}(\mathfrak{H})$. For an arbitrary vector $\mathrm{w} \in \mathcal{M}(\mathfrak{H})$ we put:

$$\mathbf{W}_{\lambda,c}[s, \mathbf{n}, J]\,\mathrm{w} := \begin{cases} \frac{\left(s\mathcal{T}(\mathrm{w}) - \frac{\lambda}{c^2}\langle \mathbf{n}, \mathrm{w}\rangle\right)}{\sqrt{\left|1 - \frac{\lambda^2}{c^2}\right|}}\mathbf{e}_0 + J\left(\frac{\lambda\mathcal{T}(\mathrm{w}) - s\langle \mathbf{n}, \mathrm{w}\rangle}{\sqrt{\left|1 - \frac{\lambda^2}{c^2}\right|}}\mathbf{n} + \mathbf{X}_1^{\perp}[\mathbf{n}]\,\mathrm{w}\right), & \lambda < \infty,\ c < \infty; \\ -\frac{\langle \mathbf{n}, \mathrm{w}\rangle}{c}\mathbf{e}_0 + J\left(c\mathcal{T}(\mathrm{w})\,\mathbf{n} + \mathbf{X}_1^{\perp}[\mathbf{n}]\,\mathrm{w}\right), & \lambda = \infty,\ c < \infty; \\ s\mathcal{T}(\mathrm{w})\mathbf{e}_0 + J\left((\lambda\mathcal{T}(\mathrm{w}) - s\langle \mathbf{n}, \mathrm{w}\rangle)\,\mathbf{n} + \mathbf{X}_1^{\perp}[\mathbf{n}]\,\mathrm{w}\right), & \lambda < \infty,\ c = \infty. \end{cases}$$

$$\mathbf{W}_{\lambda,c}[s, \mathbf{n}, J; \mathbf{a}]\mathrm{w} := \mathbf{W}_{\lambda,c}[s, \mathbf{n}, J](\mathrm{w} + \mathbf{a}). \tag{13}$$

In the case $c < \infty$ the operators of kind $\mathbf{W}_{\lambda,c}[s, \mathbf{n}, J]$ are generalized Lorentz transforms, introduced in [8] (or in the papers [5–7], for the case $\dim(\mathfrak{H}) = 3$). Under the additional conditions $\lambda < c < \infty$, $\dim(\mathfrak{H}) = 3$, $s = 1$ the formula (13) is equivalent to the formula (28b) from [19, page 43]. That is why, in this case we obtain the classical Lorentz transforms for inertial reference frame in the most general form (with arbitrary orientation of axes). Moreover, in the case $\dim(\mathfrak{H}) = 3$, $c < \infty$ the class of operators $\mathfrak{O}_+(\mathfrak{H}, c) = \{\mathbf{W}_{\lambda,c}[s, \mathbf{n}, J] \mid s = 1,\ \lambda < c\}$ coincides with the full Lorentz group in the sense of [18] (for more details see [13]). The operators of kind $\mathbf{W}_{\lambda,\infty}[s, \mathbf{n}, J]$ $(\lambda < \infty)$ are Galilean transforms (it is not difficult prove, that $\mathbf{W}_{\lambda,\infty}[s, \mathbf{n}, J] = \lim_{c\to\infty} \mathbf{W}_{\lambda,c}[s, \mathbf{n}, J]$, where the convergence is understood in the sense of uniform operator topology).

Assertion 7. *For any $c \in (0, \infty]$, $\lambda \in [0, \infty] \setminus \{c\}$, $s \in \{-1, 1\}$, $J \in \mathfrak{U}(\mathfrak{H}_1)$, $\mathbf{n} \in \mathbf{B}_1(\mathfrak{H}_1)$, and $\mathbf{a} \in \mathcal{M}(\mathfrak{H})$ it is true, that:*

$$\mathbf{W}_{\lambda,c}[s, \mathbf{n}, J; \mathbf{a}] \in \mathbf{Pk}(\mathfrak{H}).$$

Proof. Obviously it is sufficient to prove, that for $c \in (0, \infty]$, $\lambda \in [0, \infty] \setminus \{c\}$, $s \in \{-1, 1\}$, $J \in \mathfrak{U}(\mathfrak{H}_1)$ and $\mathbf{n} \in \mathbf{B}_1(\mathfrak{H}_1)$ ***the operator*** $\mathbf{W}_{\lambda,c}[s, \mathbf{n}, J]$ ***has the continuous inverse*** $\mathbf{W}_{\lambda,c}[s, \mathbf{n}, J]^{-1} \in \mathcal{L}(\mathcal{M}(\mathfrak{H}))$. For the case $c < \infty$ the highlighted statement had been proved in the paper [8].

That is why it remains to consider only the case $c = \infty$. Consider any $\lambda \in [0, \infty)$, $s \in \{-1, 1\}$, $J \in \mathfrak{U}(\mathfrak{H}_1)$ and $\mathbf{n} \in \mathbf{B}_1(\mathfrak{H}_1)$. It is easy to verify, that $\mathbf{W}_{\lambda,\infty}[s, \mathbf{n}, J] \in \mathcal{L}(\mathcal{M}(\mathfrak{H}))$. Now, we are going to prove the equality:

$$\mathbf{W}_{\lambda,\infty}[s, \mathbf{n}, J]\,\mathbf{W}_{\lambda,\infty}\left[s, J\mathbf{n}, J^{-1}\right] = \mathbb{I}_{\mathcal{M}(\mathfrak{H})}. \tag{14}$$

Chose any $\mathrm{w} \in \mathcal{M}(\mathfrak{H})$. According to (13) we have:

$$\begin{aligned} \mathbf{W}_{\lambda,\infty}[s, \mathbf{n}, J]\mathbf{W}_{\lambda,\infty}\left[s, J\mathbf{n}, J^{-1}\right]\mathrm{w} &= \mathbf{W}_{\lambda,\infty}[s, \mathbf{n}, J]\,\widetilde{\mathrm{w}}, \quad \text{where} \\ \widetilde{\mathrm{w}} = \mathbf{W}_{\lambda,\infty}\left[s, J\mathbf{n}, J^{-1}\right]\mathrm{w} &= s\mathcal{T}(\mathrm{w})\mathbf{e}_0 + \\ &+ J^{-1}\left((\lambda\mathcal{T}(\mathrm{w}) - s\langle J\mathbf{n}, \mathrm{w}\rangle)\,J\mathbf{n} + \mathbf{X}_1^{\perp}[J\mathbf{n}]\,\mathrm{w}\right). \end{aligned} \tag{15}$$

Next, applying (10), (11) and taking into account that J is the unitary operator on the subspace $\mathfrak{H}_1 \subseteq \mathcal{M}(\mathfrak{H})$, we obtain:

$$\begin{aligned} &\mathcal{T}(\widetilde{\mathrm{w}}) = s\mathcal{T}(\mathrm{w}); \\ &\langle \mathbf{n}, \widetilde{\mathrm{w}}\rangle = (\lambda\mathcal{T}(\mathrm{w}) - s\langle J\mathbf{n}, \mathrm{w}\rangle)\langle \mathbf{n}, \mathbf{n}\rangle + \left\langle \mathbf{n}, J^{-1}\mathbf{X}_1^{\perp}[J\mathbf{n}]\,\mathrm{w}\right\rangle = \\ &\quad = (\lambda\mathcal{T}(\mathrm{w}) - s\langle J\mathbf{n}, \mathrm{w}\rangle) + \left\langle J\mathbf{n}, \mathbf{X}_1^{\perp}[J\mathbf{n}]\,\mathrm{w}\right\rangle = (\lambda\mathcal{T}(\mathrm{w}) - s\langle J\mathbf{n}, \mathrm{w}\rangle); \\ &\mathbf{X}_1^{\perp}[\mathbf{n}]\,\widetilde{\mathrm{w}} = (\mathbf{X} - \mathbf{X}_1[\mathbf{n}])\,\widetilde{\mathrm{w}} = \end{aligned}$$

$$= J^{-1}\left(\left(\lambda\mathcal{T}(\mathrm{w}) - s\left\langle J\mathbf{n},\mathrm{w}\right\rangle\right) J\mathbf{n} + \mathbf{X}_1^{\perp}\left[J\mathbf{n}\right]\mathrm{w}\right) - \left\langle \mathbf{n},\widetilde{\mathrm{w}}\right\rangle \mathbf{n} =$$
$$= J^{-1}\left(\left\langle \mathbf{n},\widetilde{\mathrm{w}}\right\rangle J\mathbf{n} + \mathbf{X}_1^{\perp}\left[J\mathbf{n}\right]\mathrm{w}\right) - \left\langle \mathbf{n},\widetilde{\mathrm{w}}\right\rangle \mathbf{n} = J^{-1}\mathbf{X}_1^{\perp}\left[J\mathbf{n}\right]\mathrm{w}.$$

Herefrom, using (15), (13) and (12) we deduce:

$$\begin{aligned}
&\mathbf{W}_{\lambda,\infty}\left[s,\mathbf{n},J\right]\mathbf{W}_{\lambda,\infty}\left[s,J\mathbf{n},J^{-1}\right]\mathrm{w} = \\
&\quad = s\mathcal{T}(\widetilde{\mathrm{w}})\mathbf{e}_0 + J\left(\left(\lambda\mathcal{T}(\widetilde{\mathrm{w}}) - s\left\langle \mathbf{n},\widetilde{\mathrm{w}}\right\rangle\right)\mathbf{n} + \mathbf{X}_1^{\perp}\left[\mathbf{n}\right]\widetilde{\mathrm{w}}\right) = s\left(s\mathcal{T}(\mathrm{w})\right)\mathbf{e}_0 + \\
&\qquad + J\left(\left(\lambda\left(s\mathcal{T}(\mathrm{w})\right) - s\left(\lambda\mathcal{T}(\mathrm{w}) - s\left\langle J\mathbf{n},\mathrm{w}\right\rangle\right)\right)\mathbf{n} + J^{-1}\mathbf{X}_1^{\perp}\left[J\mathbf{n}\right]\mathrm{w}\right) = \\
&\quad = \mathcal{T}(\mathrm{w})\mathbf{e}_0 + \left\langle J\mathbf{n},\mathrm{w}\right\rangle J\mathbf{n} + \mathbf{X}_1^{\perp}\left[J\mathbf{n}\right]\mathrm{w} = \widehat{\mathbf{T}}\mathrm{w} + \mathbf{X}_1\left[J\mathbf{n}\right]\mathrm{w} + \mathbf{X}_1^{\perp}\left[J\mathbf{n}\right]\mathrm{w} = \mathrm{w}.
\end{aligned}$$

Equality (14) is proved. Applying the equality (14) to the operator $\mathbf{W}_{\lambda,\infty}\left[s,J\mathbf{n},J^{-1}\right]$, we obtain the equality: $\mathbf{W}_{\lambda,\infty}\left[s,J\mathbf{n},J^{-1}\right]\mathbf{W}_{\lambda,\infty}\left[s,\mathbf{n},J\right] = \mathbb{I}_{\mathcal{M}(\mathfrak{H})}$. Thus, $\mathbf{W}_{\lambda,c}\left[s,\mathbf{n},J\right]^{-1} = \mathbf{W}_{\lambda,\infty}\left[s,J\mathbf{n},J^{-1}\right] \in \mathcal{L}\left(\mathcal{M}\left(\mathfrak{H}\right)\right)$. □

For $0 < c \leq \infty$ we introduce the following classes of linear (non-homogeneous) operators:

$$\begin{aligned}
\mathfrak{PT}\left(\mathfrak{H},c\right) &:= \{\mathbf{W}_{\lambda,c}\left[s,\mathbf{n},J;\mathbf{a}\right] \mid s\in\{-1,1\},\ \lambda\in[0,\infty]\setminus\{c\}, \\
&\qquad \mathbf{n}\in\mathbf{B}_1\left(\mathfrak{H}_1\right),\ J\in\mathfrak{U}\left(\mathfrak{H}_1\right),\ \mathbf{a}\in\mathcal{M}\left(\mathfrak{H}\right)\}; \\
\mathfrak{PT}_+\left(\mathfrak{H},c\right) &:= \{\mathbf{W}_{\lambda,c}\left[s,\mathbf{n},J;\mathbf{a}\right]\in\mathfrak{PT}\left(\mathfrak{H},c\right) \mid s=1\}; \\
\mathfrak{P}\left(\mathfrak{H},c\right) &:= \{\mathbf{W}_{\lambda,c}\left[s,\mathbf{n},J;\mathbf{a}\right]\in\mathfrak{PT}\left(\mathfrak{H},c\right) \mid \lambda<c\}; \\
\mathfrak{P}_+\left(\mathfrak{H},c\right) &:= \{\mathbf{W}_{\lambda,c}\left[s,\mathbf{n},J;\mathbf{a}\right]\in\mathfrak{P}\left(\mathfrak{H},c\right) \mid s=1\}
\end{aligned}$$

(It is apparently, that $\mathfrak{PT}\left(\mathfrak{H},\infty\right) = \mathfrak{P}\left(\mathfrak{H},\infty\right)$, $\mathfrak{PT}_+\left(\mathfrak{H},\infty\right) = \mathfrak{P}_+\left(\mathfrak{H},\infty\right)$). Using the introduced classes of operators, we may define the following kinematic sets:

$$\begin{aligned}
\mathfrak{KPT}_0\left(\mathfrak{H},\mathcal{B},c\right) &:= \mathfrak{Kim}\left(\mathfrak{PT}\left(\mathfrak{H},c\right),\mathcal{B};\ \mathfrak{H}\right); \\
\mathfrak{KPT}\left(\mathfrak{H},\mathcal{B},c\right) &:= \mathfrak{Kim}\left(\mathfrak{PT}_+\left(\mathfrak{H},c\right),\mathcal{B};\ \mathfrak{H}\right); \\
\mathfrak{KP}_0\left(\mathfrak{H},\mathcal{B},c\right) &:= \mathfrak{Kim}\left(\mathfrak{P}\left(\mathfrak{H},c\right),\mathcal{B};\ \mathfrak{H}\right); \\
\mathfrak{KP}\left(\mathfrak{H},\mathcal{B},c\right) &:= \mathfrak{Kim}\left(\mathfrak{P}_+\left(\mathfrak{H},c\right),\mathcal{B};\ \mathfrak{H}\right).
\end{aligned}$$

In the case $\dim(\mathfrak{H}) = 3$, $c < \infty$ the kinematic set $\mathfrak{KP}\left(\mathfrak{H},\mathcal{B},c\right)$ represents the simplest mathematically strict model of the kinematics of special relativity theory in inertial frames of reference. Kinematic set $\mathfrak{KP}_0\left(\mathfrak{H},\mathcal{B},c\right)$ is constructed on the basis of general Lorentz-Poincare group, and it includes apart from usual reference frames (with positive direction of time), which have understandable physical interpretation, also reference frames with negative direction of time. Kinematic sets $\mathfrak{KPT}\left(\mathfrak{H},\mathcal{B},c\right)$ and $\mathfrak{KPT}_0\left(\mathfrak{H},\mathcal{B},c\right)$ include apart from standard ("tardyon") reference frames also "tachyon" reference frames, which are moving relatively the "tardyon" reference frames with velocity, greater than the velocity of light c. Kinematic set $\mathfrak{KP}\left(\mathfrak{H},\mathcal{B},\infty\right) = \mathfrak{KPT}\left(\mathfrak{H},\mathcal{B},\infty\right)$ in the case $\dim(\mathfrak{H}) = 3$, $c = \infty$ represents the mathematically strict model of the Galilean kinematics in the inertial frames of reference. The next corollary follows from Theorem 5.

Corollary 3. *Kinematic sets $\mathfrak{KPT}_0\left(\mathfrak{H},\mathcal{B},c\right)$, $\mathfrak{KPT}\left(\mathfrak{H},\mathcal{B},c\right)$, $\mathfrak{KP}_0\left(\mathfrak{H},\mathcal{B},c\right)$, $\mathfrak{KP}\left(\mathfrak{H},\mathcal{B},c\right)$ allow universal coordinate transform.*

Remark 10. From the results of the works [8, 13] it follows, that the sets of operators $\mathfrak{P}\left(\mathfrak{H},c\right)$ and $\mathfrak{P}_+\left(\mathfrak{H},c\right)$ form the groups of operators over the space $\mathcal{M}\left(\mathfrak{H}\right)$ (in particular case $\dim(\mathfrak{H}) = 3$ the group of operators $\mathfrak{P}_+\left(\mathfrak{H},c\right)$ coincides with the classical Poincare group in four-dimensional Minkowski space-time). At the same time, in the [9] it is proved, that the classes of operators $\mathfrak{PT}\left(\mathfrak{H},c\right)$ and $\mathfrak{PT}_+\left(\mathfrak{H},c\right)$ do not form a group over $\mathcal{M}\left(\mathfrak{H}\right)$. This means, that the kinematics $\mathfrak{KPT}_0\left(\mathfrak{H},\mathcal{B},c\right)$ and $\mathfrak{KPT}\left(\mathfrak{H},\mathcal{B},c\right)$, constructed on the basis of these classes, do not satisfy the relativity principle, because, according to Theorem 5, the subset of coordinate transforms (9), providing transition from one reference frame to all

other, is different for different frames. But, in kinematics $\mathfrak{KPT}_0(\mathfrak{H}, \mathcal{B}, c)$ and $\mathfrak{KPT}(\mathfrak{H}, \mathcal{B}, c)$ the relativity principle violated only in the superluminal diapason, because the kinematics sets $\mathfrak{KPT}_0(\mathfrak{H}, \mathcal{B}, c)$ and $\mathfrak{KPT}(\mathfrak{H}, \mathcal{B}, c)$ are formed by the "addition" of new, superlight reference frames to the kinematics sets $\mathfrak{KP}_0(\mathfrak{H}, \mathcal{B}, c)$ and $\mathfrak{KP}(\mathfrak{H}, \mathcal{B}, c)$, which satisfy the principle of relativity. It should be noted that the principle of relativity is only one of experimentally established facts. Therefore, it is possible that this principle is not satisfied when we exit out of the light barrier. Possibility of revision of the relativity principle is now discussed in the physical literature (see for example, [4, 20–25]).

7 Kinematic Sets, which do not Allow Universal Coordinate Transform.

In this section, it is constructed one interesting class of kinematic sets, in which every particle at each time moment can can have its own "velocity of light". On a physical level, the similar models (with particle-dependent velocity of light) were considered in the papers [26–30].

Let a set $\mathfrak{V}_\mathfrak{f} \subseteq (0, \infty]$ be such, that $\mathfrak{V}_\mathfrak{f} \neq \emptyset$ and $(0, \infty] \setminus \mathfrak{V}_\mathfrak{f} \neq \emptyset$. Denote:

$$\mathfrak{H}_{\mathfrak{V}_\mathfrak{f}} := \mathfrak{H} \times \mathfrak{V}_\mathfrak{f} = \{(x, c) \mid x \in \mathfrak{H},\ c \in \mathfrak{V}_\mathfrak{f}\}; \quad \mathcal{M}\left(\mathfrak{H}_{\mathfrak{V}_\mathfrak{f}}\right) := \mathbb{R} \times \mathfrak{H}_{\mathfrak{V}_\mathfrak{f}}.$$

The set $\mathcal{M}\left(\mathfrak{H}_{\mathfrak{V}_\mathfrak{f}}\right)$ will be named as the Minkowski space ***with the set of forbidden velocities*** $\mathfrak{V}_\mathfrak{f}$ over $\mathfrak{H}$. The set $\widetilde{\mathfrak{V}_\mathfrak{f}} := [0, \infty] \setminus \mathfrak{V}_\mathfrak{f}$ will be named as the ***set of allowed velocities*** for the space $\mathcal{M}\left(\mathfrak{H}_{\mathfrak{V}_\mathfrak{f}}\right)$.

For an arbitrary $\omega = (t, (x, c)) \in \mathcal{M}\left(\mathfrak{H}_{\mathfrak{V}_\mathfrak{f}}\right)$ we put $\omega^* := (t, x) \in \mathcal{M}(\mathfrak{H})$. Also for $\lambda \in \widetilde{\mathfrak{V}_\mathfrak{f}}$, $s \in \{-1, 1\}$, $J \in \mathfrak{U}(\mathfrak{H}_1)$, $\mathbf{n} \in \mathbf{B}_1(\mathfrak{H}_1)$, $\mathbf{a} \in \mathcal{M}(\mathfrak{H})$ and $\omega = (t, (x, c)) \in \mathcal{M}\left(\mathfrak{H}_{\mathfrak{V}_\mathfrak{f}}\right)$ we introduce the denotation:

$$\mathbf{W}_{\lambda;\mathfrak{V}_\mathfrak{f}}[s, \mathbf{n}, J; \mathbf{a}]\,\omega := \left(\mathsf{tm}\left(\mathbf{W}_{\lambda,c}[s, \mathbf{n}, J; \mathbf{a}]\,\omega^*\right),\ \left(\mathsf{bs}\left(\mathbf{W}_{\lambda,c}[s, \mathbf{n}, J; \mathbf{a}]\,\omega^*\right), c\right)\right). \tag{16}$$

Therefore, for any $\omega = (t, (x, c)) \in \mathcal{M}\left(\mathfrak{H}_{\mathfrak{V}_\mathfrak{f}}\right)$ we have the equality:

$$\left(\mathbf{W}_{\lambda;\mathfrak{V}_\mathfrak{f}}[s, \mathbf{n}, J; \mathbf{a}]\,\omega\right)^* = \mathbf{W}_{\lambda,c}[s, \mathbf{n}, J; \mathbf{a}]\,\omega^*. \tag{17}$$

Assertion 8. *For arbitrary* $\lambda \in \widetilde{\mathfrak{V}_\mathfrak{f}}$, $s \in \{-1, 1\}$, $J \in \mathfrak{U}(\mathfrak{H}_1)$, $\mathbf{n} \in \mathbf{B}_1(\mathfrak{H}_1)$, $\mathbf{a} \in \mathcal{M}(\mathfrak{H})$ *the mapping* $\mathbf{W}_{\lambda;\mathfrak{V}_\mathfrak{f}}[s, \mathbf{n}, J; \mathbf{a}]$ *is bijection on* $\mathcal{M}\left(\mathfrak{H}_{\mathfrak{V}_\mathfrak{f}}\right)$.

Proof. Suppose, that $\mathbf{W}_{\lambda;\mathfrak{V}_\mathfrak{f}}[s, \mathbf{n}, J; \mathbf{a}]\,\omega_1 = \mathbf{W}_{\lambda;\mathfrak{V}_\mathfrak{f}}[s, \mathbf{n}, J; \mathbf{a}]\,\omega_2$, where $\omega_1 = (t_1, (x_1, c_1)) \in \mathcal{M}\left(\mathfrak{H}_{\mathfrak{V}_\mathfrak{f}}\right)$, $\omega_2 = (t_2, (x_2, c_2)) \in \mathcal{M}\left(\mathfrak{H}_{\mathfrak{V}_\mathfrak{f}}\right)$. Then,

$$\begin{aligned}&\left(\mathsf{tm}\left(\mathbf{W}_{\lambda,c_1}[s, \mathbf{n}, J; \mathbf{a}]\,\omega_1^*\right),\ \left(\mathsf{bs}\left(\mathbf{W}_{\lambda,c_1}[s, \mathbf{n}, J; \mathbf{a}]\,\omega_1^*\right), c_1\right)\right) = \\ &\quad = \left(\mathsf{tm}\left(\mathbf{W}_{\lambda,c_2}[s, \mathbf{n}, J; \mathbf{a}]\,\omega_2^*\right),\ \left(\mathsf{bs}\left(\mathbf{W}_{\lambda,c_2}[s, \mathbf{n}, J; \mathbf{a}]\,\omega_2^*\right), c_2\right)\right).\end{aligned}$$

Consequently, $c_1 = c_2$. Hence, we have proved the equalities:

$$\begin{aligned}\mathsf{tm}\left(\mathbf{W}_{\lambda,c_1}[s, \mathbf{n}, J; \mathbf{a}]\,\omega_1^*\right) &= \mathsf{tm}\left(\mathbf{W}_{\lambda,c_1}[s, \mathbf{n}, J; \mathbf{a}]\,\omega_2^*\right)\\ \mathsf{bs}\left(\mathbf{W}_{\lambda,c_1}[s, \mathbf{n}, J; \mathbf{a}]\,\omega_1^*\right) &= \mathsf{bs}\left(\mathbf{W}_{\lambda,c_1}[s, \mathbf{n}, J; \mathbf{a}]\,\omega_2^*\right).\end{aligned}$$

Therefore, $\mathbf{W}_{\lambda,c_1}[s, \mathbf{n}, J; \mathbf{a}]\,\omega_1^* = \mathbf{W}_{\lambda,c_1}[s, \mathbf{n}, J; \mathbf{a}]\,\omega_2^*$. And, taking into account the fact, that the mapping $\mathbf{W}_{\lambda,c_1}[s, \mathbf{n}, J; \mathbf{a}]$ is bijection on $\mathcal{M}(\mathfrak{H})$, we conclude, that, $\omega_1^* = \omega_2^*$, ie $t_1 = t_2$, $x_1 = x_2$. Hence, $\omega_1 = (t_1, (x_1, c_1)) = (t_2, (x_2, c_2)) = \omega_2$. Thus, the mapping $\mathbf{W}_{\lambda;\mathfrak{V}_\mathfrak{f}}[s, \mathbf{n}, J; \mathbf{a}]$ is one-to-one correspondence.

Now it remains to prove, that $\mathbf{W}_{\lambda;\mathfrak{V}_\mathfrak{f}}[s, \mathbf{n}, J; \mathbf{a}]$ reflects the set $\mathcal{M}\left(\mathfrak{H}_{\mathfrak{V}_\mathfrak{f}}\right)$ on $\mathcal{M}\left(\mathfrak{H}_{\mathfrak{V}_\mathfrak{f}}\right)$. Consider any $\omega = (t, (x, c)) \in \mathcal{M}\left(\mathfrak{H}_{\mathfrak{V}_\mathfrak{f}}\right)$. Denote:

$$\widetilde{\omega} := \left(\mathsf{tm}\left(\left(\mathbf{W}_{\lambda,c}[s, \mathbf{n}, J; \mathbf{a}]\right)^{[-1]}\omega^*\right),\ \left(\mathsf{bs}\left(\left(\mathbf{W}_{\lambda,c}[s, \mathbf{n}, J; \mathbf{a}]\right)^{[-1]}\omega^*\right), c\right)\right).$$

Then,

$$\widetilde{\omega}^* = \left(\mathsf{tm}\left(\left(\mathbf{W}_{\lambda,c}\left[s,\mathbf{n},J;\mathbf{a}\right]\right)^{[-1]}\omega^*\right),\, \mathsf{bs}\left(\left(\mathbf{W}_{\lambda,c}\left[s,\mathbf{n},J;\mathbf{a}\right]\right)^{[-1]}\omega^*\right)\right) = \\ = \left(\mathbf{W}_{\lambda,c}\left[s,\mathbf{n},J;\mathbf{a}\right]\right)^{[-1]}\omega^*.$$

Consequently, $\mathbf{W}_{\lambda,c}\left[s,\mathbf{n},J;\mathbf{a}\right]\widetilde{\omega}^* = \omega^*$. Hence,

$$\mathbf{W}_{\lambda;\mathfrak{V}_\mathfrak{f}}\left[s,\mathbf{n},J;\mathbf{a}\right]\widetilde{\omega} = \left(\mathsf{tm}\left(\mathbf{W}_{\lambda,c}\left[s,\mathbf{n},J;\mathbf{a}\right]\widetilde{\omega}^*\right),\, \left(\mathsf{bs}\left(\mathbf{W}_{\lambda,c}\left[s,\mathbf{n},J;\mathbf{a}\right]\widetilde{\omega}^*\right),c\right)\right) = \\ = \left(\mathsf{tm}\left(\omega^*\right),\, \left(\mathsf{bs}\left(\omega^*\right),c\right)\right) = (t,(x,c)) = \omega.$$

Thus, $\mathbf{W}_{\lambda;\mathfrak{V}_\mathfrak{f}}\left[s,\mathbf{n},J;\mathbf{a}\right]$ is bijection from $\mathcal{M}\left(\mathfrak{H}_{\mathfrak{V}_\mathfrak{f}}\right)$ onto $\mathcal{M}\left(\mathfrak{H}_{\mathfrak{V}_\mathfrak{f}}\right)$. □

Denote:

$$\mathfrak{PT}\left(\mathfrak{H};\mathfrak{V}_\mathfrak{f}\right) := \Big\{\mathbf{W}_{\lambda;\mathfrak{V}_\mathfrak{f}}\left[s,\mathbf{n},J;\mathbf{a}\right] \mid \lambda\in\widetilde{\mathfrak{V}_\mathfrak{f}},\ s\in\{-1,1\}, \\ J\in\mathfrak{U}\left(\mathfrak{H}_1\right),\ \mathbf{n}\in\mathbf{B}_1\left(\mathfrak{H}_1\right),\ \mathbf{a}\in\mathcal{M}\left(\mathfrak{H}\right)\Big\}; \\ \mathfrak{PT}_+\left(\mathfrak{H};\mathfrak{V}_\mathfrak{f}\right) := \left\{\mathbf{W}_{\lambda;\mathfrak{V}_\mathfrak{f}}\left[s,\mathbf{n},J;\mathbf{a}\right]\in\mathfrak{PT}\left(\mathfrak{H};\mathfrak{V}_\mathfrak{f}\right) \mid s=1\right\}.$$

Let, $\mathcal{B}$ be a base changeable set such, that $\mathfrak{Bs}(\mathcal{B})\subseteq\mathfrak{H}_{\mathfrak{V}_\mathfrak{f}}$, $\mathbb{T}\mathbf{m}(\mathcal{B}) = (\mathbb{R},\leq)$. Then we have, $\mathbb{B}\mathfrak{s}(\mathcal{B})\subseteq\mathbb{R}\times\mathfrak{H}_{\mathfrak{V}_\mathfrak{f}} = \mathcal{M}\left(\mathfrak{H}_{\mathfrak{V}_\mathfrak{f}}\right)$. Hence, we deliver the following kinematic multi-projectors:

$$\mathfrak{PT}\left(\mathfrak{H};\mathfrak{V}_\mathfrak{f}\right)^\wedge = \left(\left((\mathbb{R},\leq),\mathfrak{H}_{\mathfrak{V}_\mathfrak{f}},\mathbf{S},\widehat{\mathfrak{H}},\mathbf{q}\right) \mid \mathbf{S}\in\mathfrak{PT}\left(\mathfrak{H};\mathfrak{V}_\mathfrak{f}\right)\right); \\ \mathfrak{PT}_+\left(\mathfrak{H};\mathfrak{V}_\mathfrak{f}\right)^\wedge = \left(\left((\mathbb{R},\leq),\mathfrak{H}_{\mathfrak{V}_\mathfrak{f}},\mathbf{S},\widehat{\mathfrak{H}},\mathbf{q}\right) \mid \mathbf{S}\in\mathfrak{PT}_+\left(\mathfrak{H};\mathfrak{V}_\mathfrak{f}\right)\right), \quad \text{where} \\ \mathbf{q}(\widetilde{x}) = x \quad \left(\forall\,\widetilde{x} = (x,c)\in\mathfrak{H}_{\mathfrak{V}_\mathfrak{f}}\right) \tag{18}$$

for $\mathcal{B}$. In accordance with Theorem 4, we can denote:

$$\mathfrak{KPT}_0\left(\mathfrak{H},\mathcal{B};\mathfrak{V}_\mathfrak{f}\right) := \mathfrak{Kim}\left[\mathfrak{PT}\left(\mathfrak{H};\mathfrak{V}_\mathfrak{f}\right)^\wedge,\mathcal{B}\right]; \\ \mathfrak{KPT}\left(\mathfrak{H},\mathcal{B};\mathfrak{V}_\mathfrak{f}\right) := \mathfrak{Kim}\left[\mathfrak{PT}_+\left(\mathfrak{H};\mathfrak{V}_\mathfrak{f}\right)^\wedge,\mathcal{B}\right].$$

It turns out, that the kinematic sets $\mathfrak{KPT}_0\left(\mathfrak{H},\mathcal{B};\mathfrak{V}_\mathfrak{f}\right)$ and $\mathfrak{KPT}\left(\mathfrak{H},\mathcal{B};\mathfrak{V}_\mathfrak{f}\right)$, in the general case, do not allow universal coordinate transform. More precisely, they allow universal coordinate transform if and only if only one value of forbidden velocity $c\in(0,\infty]$ is actually realized. In the last case, kinematics in $\mathfrak{KPT}_0\left(\mathfrak{H},\mathcal{B};\mathfrak{V}_\mathfrak{f}\right)$ or $\mathfrak{KPT}\left(\mathfrak{H},\mathcal{B};\mathfrak{V}_\mathfrak{f}\right)$ can be reduced to kinematics of type $\mathfrak{KPT}_0\left(\mathfrak{H},\mathcal{B},c\right)$ or $\mathfrak{KPT}\left(\mathfrak{H},\mathcal{B},c\right)$ (for $c<\infty$), and to Galilean kinematics (for $c=\infty$).

Theorem 6. *Let the set of forbidden velocities $\mathfrak{V}_\mathfrak{f}$ be separated from zero (ie there exists a number $\eta>0$ such, that $\mathfrak{V}_\mathfrak{f}\subseteq[\eta,\infty]$).*

Kinematic set $\mathfrak{KPT}\left(\mathfrak{H},\mathcal{B};\mathfrak{V}_\mathfrak{f}\right)$ allows universal coordinate transform if and only if there don't exist elementary states $\widetilde{x}_1 = (x_1,c_1)$, $\widetilde{x}_2 = (x_2,c_2)\in\mathfrak{Bs}(\mathcal{B})$ such, that $c_1\neq c_2$.

To prove Theorem 6 we need the following two lemmas.

Lemma 1. *Chose any fixed $c_1,c_2\in(0,\infty]$, $c_1\neq c_2$, $s\in\{-1,1\}$ and $J\in\mathfrak{U}\left(\mathfrak{H}_1\right)$.*

Then, for any number $\varepsilon\in(0,\min(c_1,c_2))$, and arbitrary vectors $\mathrm{w}_1,\mathrm{w}_2\in\mathcal{M}\left(\mathfrak{H}\right)$ such, that $\mathrm{w}_1\neq\mathrm{w}_2$ there exist $\lambda\in(0,\varepsilon)$, $\mathbf{n}\in\mathbf{B}_1\left(\mathfrak{H}_1\right)$ and $\mathbf{a}\in\mathcal{M}\left(\mathfrak{H}\right)$, for which the following equality holds:

$$\mathbf{W}_{\lambda,c_1}\left[s,\mathbf{n},J;\mathbf{a}\right]\mathrm{w}_1 = \mathbf{W}_{\lambda,c_2}\left[s,\mathbf{n},J;\mathbf{a}\right]\mathrm{w}_2.$$

Proof. Further, for convenience, we assume, that $c_1<c_2$. Obviously, this assumption does not restrict the the generality of our conclusions.

1. At first, we are going to prove Lemma in the special case $\mathrm{w}_1 = \mathbf{0}$, $\mathrm{w}_2 = \mathrm{w} \neq \mathbf{0}$. Consider any, $\varepsilon \in (0, \min(c_1, c_2))$. According to the specifics of this case, we should find $\lambda \in (0, \varepsilon)$, $\mathbf{n} \in \mathbf{B}_1(\mathfrak{H}_1)$ and $\mathbf{a} \in \mathcal{M}(\mathfrak{H})$, such, that:

$$\mathbf{W}_{\lambda,c_1}[s, \mathbf{n}, J; \mathbf{a}]\,\mathbf{0} = \mathbf{W}_{\lambda,c_2}[s, \mathbf{n}, J; \mathbf{a}]\,\mathrm{w}. \tag{19}$$

Taking into account (13), we can rewrite the last condition in the form:

$$\mathbf{W}_{\lambda,c_1}[s, \mathbf{n}, J]\,\mathbf{a} = \mathbf{W}_{\lambda,c_2}[s, \mathbf{n}, J]\,(\mathrm{w} + \mathbf{a}). \tag{20}$$

Denote:

$$t := \mathcal{T}(\mathrm{w}), \qquad x := \mathbf{X}\mathrm{w}. \tag{21}$$

Then we can write, $\mathrm{w} = t\mathbf{e}_0 + x$.

Consider any fixed vector $\mathbf{n}_0 \in \mathbf{B}_1(\mathfrak{H}_1)$. Denote:

$$\mathbf{n} := \begin{cases} \frac{x}{\|x\|}, & x \neq \mathbf{0} \\ \mathbf{n}_0, & x = \mathbf{0}. \end{cases} \tag{22}$$

Then, we have:

$$\begin{aligned} x &= \|x\|\,\mathbf{n}, \\ \langle \mathbf{n}, \mathrm{w} \rangle &= \langle \mathbf{n}, x \rangle = \|x\|, \\ \mathbf{X}_1^{\perp}[\mathbf{n}]\,\mathrm{w} &= \mathbf{X}\mathrm{w} - \langle \mathbf{n}, \mathrm{w} \rangle\,\mathbf{n} = x - \|x\|\,\mathbf{n} = x - x = \mathbf{0}. \end{aligned} \tag{23}$$

Vector $\mathbf{a}$ we seek in the form:

$$\mathbf{a} = \tau\mathbf{e}_0 + \mu\mathbf{n}, \qquad \text{where} \quad \tau, \mu \in \mathbb{R}. \tag{24}$$

1.a) At first we consider the case $c_1, c_2 < \infty$.

Substituting the value of the vector $\mathbf{a}$ from (24) into the condition (20) and applying (21), (23), (13), we obtain the following condition:

$$\begin{aligned} &\left(s\tau - \frac{\lambda}{c_1^2}\mu\right)\gamma\left(\frac{\lambda}{c_1}\right)\mathbf{e}_0 + (\lambda\tau - s\mu)\,\gamma\left(\frac{\lambda}{c_1}\right) J\mathbf{n} = \\ &\quad = \left(s(t+\tau) - \frac{\lambda}{c_2^2}(\|x\| + \mu)\right)\gamma\left(\frac{\lambda}{c_2}\right)\mathbf{e}_0 + \\ &\qquad + (\lambda(t+\tau) - s(\|x\| + \mu))\,\gamma\left(\frac{\lambda}{c_2}\right) J\mathbf{n}, \\ &\qquad\qquad \text{where} \quad \gamma(\xi) = \frac{1}{\sqrt{|1-\xi^2|}}, \quad \xi \geq 0,\ \xi \neq 1. \end{aligned} \tag{25}$$

Taking into account orthogonality of the vector $\mathbf{e}_0$ to the subspace $\mathfrak{H}_1$ and unitarity of the operator J on the subspace $\mathfrak{H}_1$, we get the following system of equations:

$$\begin{cases} \left(s\tau - \frac{\lambda}{c_1^2}\mu\right)\gamma\left(\frac{\lambda}{c_1}\right) = \left(s(t+\tau) - \frac{\lambda}{c_2^2}(\|x\| + \mu)\right)\gamma\left(\frac{\lambda}{c_2}\right) \\ (\lambda\tau - s\mu)\,\gamma\left(\frac{\lambda}{c_1}\right) = (\lambda(t+\tau) - s(\|x\| + \mu))\,\gamma\left(\frac{\lambda}{c_2}\right) \end{cases} \tag{26}$$

By means of simple transformations, the system (26) can be reduced to the form:

$$\begin{cases} \tau\left(\gamma\left(\frac{\lambda}{c_2}\right) - \gamma\left(\frac{\lambda}{c_1}\right)\right) = \lambda s\left(\left(\frac{\|x\|+\mu}{c_2^2}\right)\gamma\left(\frac{\lambda}{c_2}\right) - \frac{\mu}{c_1^2}\gamma\left(\frac{\lambda}{c_1}\right)\right) - t\gamma\left(\frac{\lambda}{c_2}\right) \\ \lambda\tau\left(\gamma\left(\frac{\lambda}{c_2}\right) - \gamma\left(\frac{\lambda}{c_1}\right)\right) = s\left((\|x\| + \mu)\,\gamma\left(\frac{\lambda}{c_2}\right) - \mu\gamma\left(\frac{\lambda}{c_1}\right)\right) - \lambda t\gamma\left(\frac{\lambda}{c_2}\right) \end{cases} \tag{27}$$

Replacing the expression $\tau\left(\gamma\left(\frac{\lambda}{c_2}\right)-\gamma\left(\frac{\lambda}{c_1}\right)\right)$ in the second equation of the system (27) by the right-hand side of the first equation of this system, we deliver the equation:

$$\lambda^2\left(\frac{\|x\|+\mu}{c_2^2}\gamma\left(\frac{\lambda}{c_2}\right)-\frac{\mu}{c_1^2}\gamma\left(\frac{\lambda}{c_1}\right)\right)=\left(\|x\|+\mu\right)\gamma\left(\frac{\lambda}{c_2}\right)-\mu\gamma\left(\frac{\lambda}{c_1}\right).$$

After a simple transformations the last equation takes a form:

$$\frac{\left(\|x\|+\mu\right)\left(1-\frac{\lambda^2}{c_2^2}\right)}{\sqrt{\left|1-\frac{\lambda^2}{c_2^2}\right|}}-\frac{\mu\left(1-\frac{\lambda^2}{c_1^2}\right)}{\sqrt{\left|1-\frac{\lambda^2}{c_1^2}\right|}}=0. \tag{28}$$

Now, we introduce the denotations:

$$\Phi_1(y):=\operatorname{sign}(y)\sqrt{|y|};\quad \Phi_2(y)=y\,|y|\qquad (y\in\mathbb{R}). \tag{29}$$

In the case $y\neq 0$ the function $\Phi_1(y)$ may be represented in the form, $\Phi_1(y)=\frac{y}{\sqrt{|y|}}$.

In view of denotation (29) the equation (28) becomes:

$$\Phi_1\left(\left(\|x\|+\mu\right)\left|\|x\|+\mu\right|\left(1-\frac{\lambda^2}{c_2^2}\right)\right)=\Phi_1\left(\mu\,|\mu|\left(1-\frac{\lambda^2}{c_1^2}\right)\right).$$

Taking into account, that the function Φ_1 is strictly monotone on $\mathbb{R}$, we get the equation:

$$\left(\|x\|+\mu\right)\left|\|x\|+\mu\right|\left(1-\frac{\lambda^2}{c_2^2}\right)=\mu\,|\mu|\left(1-\frac{\lambda^2}{c_1^2}\right),$$

which after a simple transformations is reduced to the form:

$$\lambda^2\left(\Phi_2\left(\frac{\|x\|+\mu}{c_2}\right)-\Phi_2\left(\frac{\mu}{c_1}\right)\right)=\Phi_2\left(\|x\|+\mu\right)-\Phi_2\left(\mu\right). \tag{30}$$

Since $c_1<c_2$, then for $\mu<-\|x\|$ we have $\frac{\|x\|+\mu}{c_2}>\frac{\mu}{c_1}$. Therefore, taking into account, that the function Φ_2 is strictly increasing on $\mathbb{R}$, we may define the function:

$$\Phi_{3;x}(\mu)=\sqrt{\frac{\Phi_2\left(\|x\|+\mu\right)-\Phi_2\left(\mu\right)}{\Phi_2\left(\frac{\|x\|+\mu}{c_2}\right)-\Phi_2\left(\frac{\mu}{c_1}\right)}}=\sqrt{\frac{\mu^2-\left(\|x\|+\mu\right)^2}{\left(\frac{\mu}{c_1}\right)^2-\left(\frac{\|x\|+\mu}{c_2}\right)^2}},\qquad \mu<-\|x\|.$$

It is easy to verify, that $\Phi_{3;x}(\mu)\to 0$, $\mu\to-\infty$. Hence, there exists the number $\mu_0<-\|x\|$ such, that $\Phi_{3;x}(\mu_0)\in[0,\varepsilon)$.

In the case $x\neq\mathbf{0}$ we have $\Phi_{3;x}(\mu)>0$ for all μ such, that $\mu<-\|x\|$. In the case $x=\mathbf{0}$, the equation (30) becomes the true equality for $\mu=0$ and arbitrary $\lambda\in\mathbb{R}$. That is why, if we put:

$$\mu:=\begin{cases}\mu_0, & x\neq\mathbf{0}\\ 0, & x=\mathbf{0}\end{cases};\qquad \lambda:=\begin{cases}\Phi_{3;x}(\mu_0), & x\neq\mathbf{0}\\ \frac{\varepsilon}{2}, & x=\mathbf{0},\end{cases} \tag{31}$$

we will obtain the values $\mu\in\mathbb{R}$ and $\lambda\in(0,\varepsilon)$, for which the equality (30) holds.

Since $0<\lambda<\varepsilon<\min(c_1,c_2)$, then for values λ,μ, determined by the formula (31), the second equation from the system (27) takes the form:

$$\lambda\tau\left(\frac{1}{\sqrt{1-\frac{\lambda^2}{c_2^2}}}-\frac{1}{\sqrt{1-\frac{\lambda^2}{c_1^2}}}\right)=s\left(\frac{\|x\|+\mu}{\sqrt{1-\frac{\lambda^2}{c_2^2}}}-\frac{\mu}{\sqrt{1-\frac{\lambda^2}{c_1^2}}}\right)-\frac{\lambda t}{\sqrt{1-\frac{\lambda^2}{c_2^2}}}, \tag{32}$$

where, considering that $\lambda > 0$ and $c_1 < c_2$, we have $\lambda \left(\frac{1}{\sqrt{1-\frac{\lambda^2}{c_2^2}}} - \frac{1}{\sqrt{1-\frac{\lambda^2}{c_1^2}}} \right) \neq 0$. Hence, the number τ is uniquely determined by the equality (32). Then, the vector $\mathbf{a}$ we calculate by the formula (24). And, substituting the delivered values of the parameters $\lambda \in (0, \varepsilon)$, $\mathbf{n} \in \mathbf{B}_1(\mathfrak{H}_1)$ and $\mathbf{a} \in \mathcal{M}(\mathfrak{H})$ into (20), we guarantee the valid equality. In the case $c_2, c_2 < \infty$ and $\mathrm{w}_1 = \mathbf{0}$, Lemma is proved.

1.b) Thus, it remains to consider only the case $c_2 = \infty$, $c_1 < \infty$ ($\mathrm{w}_1 = \mathbf{0}$, $\mathrm{w}_2 = \mathrm{w} \neq \mathbf{0}$). Note, that the case $c_1 = \infty$ is impossible, because $c_1 < c_2$.

Substituting the value of the vector $\mathbf{a}$ from (24) into the condition (20) and applying (21), (23), (13), we obtain the following condition:

$$\left(s\tau - \frac{\lambda}{c_1^2}\mu \right) \gamma \left(\frac{\lambda}{c_1} \right) \mathbf{e}_0 + (\lambda\tau - s\mu) \gamma \left(\frac{\lambda}{c_1} \right) J\mathbf{n} = \\ = s(t+\tau)\mathbf{e}_0 + (\lambda(t+\tau) - s(\|x\| + \mu))\, J\mathbf{n}.$$

Hence, taking into account orthogonality of the vector $\mathbf{e}_0$ to the subspace $\mathfrak{H}_1$ and unitarity of the operator J on the subspace $\mathfrak{H}_1$, we get the following system of equations:

$$\begin{cases} \left(s\tau - \frac{\lambda}{c_1^2}\mu \right) \gamma \left(\frac{\lambda}{c_1} \right) = s(t+\tau) \\ (\lambda\tau - s\mu) \gamma \left(\frac{\lambda}{c_1} \right) = \lambda(t+\tau) - s(\|x\| + \mu) \end{cases} \tag{33}$$

After a simple transformations, the system (33) may be reduced to the form:

$$\begin{cases} \tau \left(1 - \gamma \left(\frac{\lambda}{c_1} \right) \right) = -\lambda s \frac{\mu}{c_1^2} \gamma \left(\frac{\lambda}{c_1} \right) - t \\ \lambda\tau \left(1 - \gamma \left(\frac{\lambda}{c_1} \right) \right) = s \left(\|x\| + \mu - \mu\gamma \left(\frac{\lambda}{c_1} \right) \right) - \lambda t \end{cases} \tag{34}$$

Replacing the expression $\tau \left(1 - \gamma \left(\frac{\lambda}{c_1} \right) \right)$ in the second equation of the system (34) by the right-hand side of the first equation of this system, we obtain the equation:

$$-\lambda^2 \frac{\mu}{c_1^2} \gamma \left(\frac{\lambda}{c_1} \right) = \|x\| + \mu - \mu\gamma \left(\frac{\lambda}{c_1} \right),$$

which, by means of a simple transformations takes a form:

$$\Phi_1 \left((\|x\| + \mu) \left| \|x\| + \mu \right| \right) = \Phi_1 \left(\mu |\mu| \left(1 - \frac{\lambda^2}{c_1^2} \right) \right), \tag{35}$$

where the function Φ_1 is determined by the formula (29). Taking into account, that the function Φ_1 is strictly monotone on $\mathbb{R}$, we get the equation:

$$(\|x\| + \mu) \left| \|x\| + \mu \right| = \mu |\mu| \left(1 - \frac{\lambda^2}{c_1^2} \right),$$

which after a simple transformations is reduced to the form:

$$-\lambda^2 \Phi_2 \left(\frac{\mu}{c_1} \right) = \Phi_2 (\|x\| + \mu) - \Phi_2(\mu). \tag{36}$$

Therefore, taking into account, that the function $\Phi_2(y) = y|y|$ is strictly increasing on $\mathbb{R}$, we may define the function:

$$\Phi_{3;x}^{\infty}(\mu) = \sqrt{\frac{\Phi_2(\|x\| + \mu) - \Phi_2(\mu)}{-\Phi_2\left(\frac{\mu}{c_1} \right)}} = \sqrt{\frac{\mu^2 - (\|x\| + \mu)^2}{\left(\frac{\mu}{c_1} \right)^2}}, \qquad \mu < -\|x\|.$$

It is easy to verify, that $\Phi_{3;x}^{\infty}(\mu) \to 0$, $\mu \to -\infty$. Hence, there exists the number $\mu_0 < -\|x\|$ such, that $\Phi_{3;x}^{\infty}(\mu_0) \in [0, \varepsilon)$.

In the case $x \neq \mathbf{0}$ we have $\Phi_{3;x}^{\infty}(\mu) > 0$ for all μ such, that $\mu < -\|x\|$. In the case $x = \mathbf{0}$, the equation (36) becomes the true equality for $\mu = 0$ and arbitrary $\lambda \in \mathbb{R}$. That is why, if we put:

$$\mu := \begin{cases} \mu_0, & x \neq \mathbf{0} \\ 0, & x = \mathbf{0} \end{cases} \qquad \lambda := \begin{cases} \Phi_{3;x}^{\infty}(\mu_0), & x \neq \mathbf{0} \\ \frac{\varepsilon}{2}, & x = \mathbf{0}, \end{cases} \tag{37}$$

we will obtain the values $\mu \in \mathbb{R}$ and $\lambda \in (0, \varepsilon)$, for which the equality (36) is true.

Since $0 < \lambda < \varepsilon < \min(c_1, c_2) = c_1$, then for values λ, μ, determined by the formula (37), the second equation from the system (34) may be rewritten in the form:

$$\lambda\tau \left(1 - \frac{1}{\sqrt{1 - \frac{\lambda^2}{c_1^2}}}\right) = s\left(\|x\| + \mu - \frac{\mu}{\sqrt{1 - \frac{\lambda^2}{c_1^2}}}\right) - \lambda t, \tag{38}$$

where, considering that $\lambda, c_1 > 0$, we have, $\lambda\left(1 - \frac{1}{\sqrt{1 - \frac{\lambda^2}{c_1^2}}}\right) \neq 0$. Hence, the number τ is uniquely determined by the equality (38). Then, the vector $\mathbf{a}$ we calculate by the formula (24). And, substituting the delivered values of the parameters $\lambda \in (0, \varepsilon)$, $\mathbf{n} \in \mathbf{B}_1(\mathfrak{H}_1)$ and $\mathbf{a} \in \mathcal{M}(\mathfrak{H})$ into (20), we obtain the valid equality. Hence, in the case $c_1 < \infty$, $c_2 = \infty$ and $\mathrm{w}_1 = \mathbf{0}$, Lemma is proved.

2. We now turn to the general case, where $\mathrm{w}_1, \mathrm{w}_2$ are arbitrary vectors of the space $\mathcal{M}(\mathfrak{H})$ such, that $\mathrm{w}_1 \neq \mathrm{w}_2$. According to the result, proved in the first item of Lemma, parameters $\lambda \in (0, \varepsilon)$, $\mathbf{n} \in \mathbf{B}_1(\mathfrak{H}_1)$ and $\widetilde{\mathbf{a}} \in \mathcal{M}(\mathfrak{H})$, exist such, that $\mathbf{W}_{\lambda,c_1}[s, \mathbf{n}, J]\widetilde{\mathbf{a}} = \mathbf{W}_{\lambda,c_2}[s, \mathbf{n}, J](\mathrm{w}_2 - \mathrm{w}_1 + \widetilde{\mathbf{a}})$. Denote, $\mathbf{a} := \widetilde{\mathbf{a}} - \mathrm{w}_1$. Then, taking into account, (13), we receive the desired equality $\mathbf{W}_{\lambda,c_1}[s, \mathbf{n}, J; \mathbf{a}]\,\mathrm{w}_1 = \mathbf{W}_{\lambda,c_2}[s, \mathbf{n}, J; \mathbf{a}]\,\mathrm{w}_2$. □

For $y_1, y_2 \in (0, \infty]$, such, that $y_1, y_2 \neq \infty$ in the case $y_1 = y_2$ we put:

$$\sigma(y_1, y_2) = \begin{cases} \left(\frac{y_1^{-2} + y_2^{-2}}{2}\right)^{-\frac{1}{2}}, & y_1, y_2 < \infty \\ \sqrt{2}\, y_1, & y_1 < \infty,\ y_2 = \infty \\ \sqrt{2}\, y_2, & y_1 = \infty,\ y_2 < \infty \end{cases} \cdot \tag{39}$$

Lemma 2. *Suppose, that for some vector* $\mathrm{w} \in \mathcal{M}(\mathfrak{H})$ *it holds the equality*

$$\mathbf{W}_{\lambda,c_1}[s, \mathbf{n}, J]\,\mathrm{w} = \mathbf{W}_{\lambda,c_2}[s, \mathbf{n}, J]\,\mathrm{w},$$

where $c_1, c_2 \in (0, \infty]$, $\lambda \in (0, \infty] \setminus \{c_1, c_2, \sigma(c_1, c_2)\}$, $s \in \{-1, 1\}$, $J \in \mathfrak{U}(\mathfrak{H}_1)$, $\mathbf{n} \in \mathbf{B}_1(\mathfrak{H}_1)$ *with* $c_1 \neq c_2$. *Then,* $\mathcal{T}(\mathrm{w}) = \langle \mathbf{n}, \mathrm{w} \rangle = 0$.

Proof of Lemma we divide into a few cases.

Case 1: $c_1, c_2 < \infty$, $\lambda < \infty$. In this case, by the formula (13), we get

$$\begin{aligned} &\mathbf{W}_{\lambda,c_1}[s, \mathbf{n}, J]\,\mathrm{w} - \mathbf{W}_{\lambda,c_2}[s, \mathbf{n}, J]\,\mathrm{w} = \\ &= \left(\left(\gamma\left(\frac{\lambda}{c_1}\right) - \gamma\left(\frac{\lambda}{c_2}\right)\right) s\mathcal{T}(\mathrm{w}) - \left(\frac{\lambda}{c_1^2}\gamma\left(\frac{\lambda}{c_1}\right) - \frac{\lambda}{c_2^2}\gamma\left(\frac{\lambda}{c_2}\right)\right)\langle \mathbf{n}, \mathrm{w} \rangle\right)\mathbf{e}_0 + \\ &\quad + \left(\gamma\left(\frac{\lambda}{c_1}\right) - \gamma\left(\frac{\lambda}{c_2}\right)\right)(\lambda\mathcal{T}(\mathrm{w}) - s\langle \mathbf{n}, \mathrm{w} \rangle)\, J\mathbf{n}, \end{aligned} \tag{40}$$

where the function $\gamma : [0,\infty) \mapsto \mathbb{R}$ is determined by the formula (25). By conditions of Lemma, $\mathbf{W}_{\lambda,c_1}[s,\mathbf{n},J]\,\mathrm{w} - \mathbf{W}_{\lambda,c_2}[s,\mathbf{n},J]\,\mathrm{w} = \mathbf{0}$, where $\mathbf{0}$ is zero vector of the space $\mathcal{M}(\mathfrak{H})$. Hence, the right-hand side of the equality (40) is equal to zero vector. Therefore, taking into account orthogonality of the vector $\mathbf{e}_0$ to the subspace $\mathfrak{H}_1$ and unitarity of the operator J on the subspace $\mathfrak{H}_1$, we get the following equalities:

$$\begin{aligned} s\left(\gamma\left(\tfrac{\lambda}{c_1}\right)-\gamma\left(\tfrac{\lambda}{c_2}\right)\right)\mathcal{T}(\mathrm{w}) - \left(\tfrac{\lambda}{c_1^2}\gamma\left(\tfrac{\lambda}{c_1}\right)-\tfrac{\lambda}{c_2^2}\gamma\left(\tfrac{\lambda}{c_2}\right)\right)\langle \mathbf{n},\mathrm{w}\rangle &= 0;\\ \left(\gamma\left(\tfrac{\lambda}{c_1}\right)-\gamma\left(\tfrac{\lambda}{c_2}\right)\right)\ \left(\lambda\mathcal{T}(\mathrm{w}) - s\langle \mathbf{n},\mathrm{w}\rangle\right) &= 0. \end{aligned} \tag{41}$$

According to the conditions of Lemma, $\lambda > 0$ and $\lambda \neq \sigma(c_1,c_2) = \sqrt{\frac{2}{\frac{1}{c_1^2}+\frac{1}{c_2^2}}}$. Consequently, $\gamma\left(\frac{\lambda}{c_1}\right)-\gamma\left(\frac{\lambda}{c_2}\right) \neq 0$. Thus, the equalities (41) may be rewritten in the form:

$$\left\{\begin{aligned} s\left(\gamma\left(\tfrac{\lambda}{c_1}\right)-\gamma\left(\tfrac{\lambda}{c_2}\right)\right)\mathcal{T}(\mathrm{w}) - \left(\tfrac{\lambda}{c_1^2}\gamma\left(\tfrac{\lambda}{c_1}\right)-\tfrac{\lambda}{c_2^2}\gamma\left(\tfrac{\lambda}{c_2}\right)\right)\langle \mathbf{n},\mathrm{w}\rangle &= 0;\\ \lambda\mathcal{T}(\mathrm{w}) - s\langle \mathbf{n},\mathrm{w}\rangle &= 0. \end{aligned}\right. \tag{42}$$

The system (42) is a system of linear homogeneous equations relatively the variables $\mathcal{T}(\mathrm{w})$ and $\langle \mathbf{n},\mathrm{w}\rangle$. Determinant of this system is:

$$\begin{aligned} \Delta &= -\left[\left(\gamma\left(\frac{\lambda}{c_1}\right)-\gamma\left(\frac{\lambda}{c_2}\right)\right) - \left(\frac{\lambda^2}{c_1^2}\gamma\left(\frac{\lambda}{c_1}\right)-\frac{\lambda^2}{c_2^2}\gamma\left(\frac{\lambda}{c_2}\right)\right)\right] = \\ &= -\left(\boldsymbol{g}\left(\frac{\lambda}{c_1}\right)-\boldsymbol{g}\left(\frac{\lambda}{c_2}\right)\right), \qquad \text{where} \\ &\boldsymbol{g}(\xi) = \left(1-\xi^2\right)\gamma(\xi) = \operatorname{sign}\,(1-\xi)\sqrt{|1-\xi^2|} \qquad (\xi\geq 0,\ \xi\neq 1). \end{aligned}$$

Since the function $\boldsymbol{g}(\xi) = \operatorname{sign}\,(1-\xi)\sqrt{|1-\xi^2|}$ is strictly decreasing on $[0,\infty)$, determinant Δ of the system (42) is nonzero. Hence, $\mathcal{T}(\mathrm{w}) = \langle \mathbf{n},\mathrm{w}\rangle = 0$, that was necessary to prove.

Case 2: $c_1, c_2 < \infty$, $\lambda = \infty$.

In this case, by the formula (13), we receive:

$$\begin{aligned} 0 &= \mathbf{W}_{\lambda,c_1}[s,\mathbf{n},J]\,\mathrm{w} - \mathbf{W}_{\lambda,c_2}[s,\mathbf{n},J]\,\mathrm{w} = \\ &= -\frac{\langle \mathbf{n},\mathrm{w}\rangle}{c_1}\mathbf{e}_0 + c_1\mathcal{T}(\mathrm{w})\,J\mathbf{n} - \left(-\frac{\langle \mathbf{n},\mathrm{w}\rangle}{c_2}\mathbf{e}_0 + c_2\mathcal{T}(\mathrm{w})\,J\mathbf{n}\right) = \\ &= -\left(\frac{1}{c_1}-\frac{1}{c_2}\right)\langle \mathbf{n},\mathrm{w}\rangle\,\mathbf{e}_0 + (c_1-c_2)\,\mathcal{T}(\mathrm{w})\,J\mathbf{n}. \end{aligned}$$

And since $c_1 \neq c_2$, taking into account orthogonality of the vector $\mathbf{e}_0$ to the subspace $\mathfrak{H}_1$ and unitarity of the operator J on the subspace $\mathfrak{H}_1$, we get the equality $\mathcal{T}(\mathrm{w}) = \langle \mathbf{n},\mathrm{w}\rangle = 0$.

Case 3: $c_1 < \infty$, $c_2 = \infty$.

By the conditions of Lemma $\lambda \neq c_2$. Hence, in this case we have $\lambda < \infty$. And, according to (13), we obtain:

$$\begin{aligned} 0 &= \mathbf{W}_{\lambda,c_1}[s,\mathbf{n},J]\,\mathrm{w} - \mathbf{W}_{\lambda,c_2}[s,\mathbf{n},J]\,\mathrm{w} = \\ &= \left(\left(\gamma\left(\frac{\lambda}{c_1}\right)-1\right)s\mathcal{T}(\mathrm{w}) - \frac{\lambda}{c_1^2}\gamma\left(\frac{\lambda}{c_1}\right)\langle \mathbf{n},\mathrm{w}\rangle\right)\mathbf{e}_0 + \\ &\quad + \left(\gamma\left(\frac{\lambda}{c_1}\right)-1\right)\left(\lambda\mathcal{T}(\mathrm{w}) - s\langle \mathbf{n},\mathrm{w}\rangle\right)J\mathbf{n}. \end{aligned} \tag{43}$$

By the conditions of Lemma, $\lambda > 0$ and $\lambda \neq \sigma(c_1,c_2) = \sqrt{2}\,c_1$. Thus, $\gamma\left(\frac{\lambda}{c_1}\right)-1 \neq 0$. Hence, taking into account orthogonality of the vector $\mathbf{e}_0$ to the subspace $\mathfrak{H}_1$ and unitarity of the

operator J on the subspace $\mathfrak{H}_1$, from the equality (43) we receive the system of equations:

$$\left\{\begin{array}{rl}\left(\gamma\left(\frac{\lambda}{c_1}\right)-1\right)s\mathcal{T}(\mathrm{w})-\frac{\lambda}{c_1^2}\gamma\left(\frac{\lambda}{c_1}\right)\langle\mathbf{n},\mathrm{w}\rangle & =0\\ \lambda\mathcal{T}(\mathrm{w})-s\langle\mathbf{n},\mathrm{w}\rangle & =0.\end{array}\right. \tag{44}$$

The system (44) is a system of linear homogeneous equations relatively the variables $\mathcal{T}(\mathrm{w})$ and $\langle\mathbf{n},\mathrm{w}\rangle$. Determinant of this system is:

$$\Delta_1=-\left(\left(\gamma\left(\frac{\lambda}{c_1}\right)-1\right)-\frac{\lambda^2}{c_1^2}\gamma\left(\frac{\lambda}{c_1}\right)\right)=-\left(\boldsymbol{g}\left(\frac{\lambda}{c_1}\right)-\boldsymbol{g}(0)\right).$$

Since, by the conditions of Lemma, $\lambda>0$ and $c_1<\infty$, then $\frac{\lambda}{c_1}\neq 0$. That is why, $\Delta_1\neq 0$. Thus, $\mathcal{T}(\mathrm{w})=\langle\mathbf{n},\mathrm{w}\rangle=0$.

Case 4: $c_1=\infty$, $c_2<\infty$ is considered similarly to the case 3.

Case $c_1,c_2=\infty$ is impossible, because, by the conditions of Lemma, $c_1\neq c_2$. $\square$

Corollary 4. *Let,* $c_1,c_2\in(0,\infty]$, $c_1\neq c_2$, $s\in\{-1,1\}$, $J\in\mathfrak{U}(\mathfrak{H}_1)$, $\mathbf{n}\in\mathbf{B}_1(\mathfrak{H}_1)$.

Then for any $\mathrm{w}\in\mathcal{M}(\mathfrak{H})$ *and* $\varepsilon\in(0,\min(c_1,c_2))$ *there exist* $\lambda\in(0,\varepsilon)$ *and* $\mathbf{a}\in\mathcal{M}(\mathfrak{H})$, *such, that*

$$\mathbf{W}_{\lambda,c_1}[s,\mathbf{n},J;\mathbf{a}]\,\mathrm{w}\neq\mathbf{W}_{\lambda,c_2}[s,\mathbf{n},J;\mathbf{a}]\,\mathrm{w},$$

Proof. Let us chose any $\mathbf{a}\in\mathcal{M}(\mathfrak{H})$ such, that:

$$\mathcal{T}(\mathrm{w}+\mathbf{a})\neq 0. \tag{45}$$

Also we chose any number $\lambda\in(0,\varepsilon)\setminus\{\sigma(c_1,c_2)\}$. If we assume, that $\mathbf{W}_{\lambda,c_1}[s,\mathbf{n},J;\mathbf{a}]\,\mathrm{w}=\mathbf{W}_{\lambda,c_2}[s,\mathbf{n},J;\mathbf{a}]\,\mathrm{w}$, then, according to (13), we will obtain:

$$\mathbf{W}_{\lambda,c_1}[s,\mathbf{n},J](\mathrm{w}+\mathbf{a})=\mathbf{W}_{\lambda,c_2}[s,\mathbf{n},J](\mathrm{w}+\mathbf{a}).$$

Hence, by Lemma 2, $\mathcal{T}(\mathrm{w}+\mathbf{a})=0$, contrary to the correlation (45). Thus, $\mathbf{W}_{\lambda,c_1}[s,\mathbf{n},J;\mathbf{a}]\,\mathrm{w}\neq\mathbf{W}_{\lambda,c_2}[s,\mathbf{n},J;\mathbf{a}]\,\mathrm{w}$. $\square$

Proof of Theorem 6. **1**. For any fixed vector $\mathbf{n}\in\mathbf{B}_1(\mathfrak{H}_1)$ we are going to prove the equality:

$$\mathbf{W}_{0;\mathfrak{V}_\mathfrak{f}}[1,\mathbf{n},\mathbb{I}_{-1,1}[\mathbf{n}],\mathbf{0}]=\mathbb{I}_{\mathcal{M}\left(\mathfrak{H}_{\mathfrak{V}_\mathfrak{f}}\right)}, \tag{46}$$

where $\mathbb{I}_{\mathcal{M}\left(\mathfrak{H}_{\mathfrak{V}_\mathfrak{f}}\right)}$ is the the identity operator on $\mathcal{M}\left(\mathfrak{H}_{\mathfrak{V}_\mathfrak{f}}\right)$, and

$$\mathbb{I}_{\kappa,\mu}[\mathbf{n}]\,x:=\kappa\mathbf{X}_1[\mathbf{n}]\,x+\mu\mathbf{X}_1^{\perp}[\mathbf{n}]\,x,\quad x\in\mathfrak{H}_1\qquad(\mathbf{n}\in\mathbf{B}_1(\mathfrak{H}_1),\ \kappa,\mu\in\{-1,1\}).$$

Indeed, according to (16), (13), for an arbitrary element $\omega=(t,(x,c))\in\mathcal{M}\left(\mathfrak{H}_{\mathfrak{V}_\mathfrak{f}}\right)$, we have:

$$\begin{aligned}&\mathbf{W}_{0;\mathfrak{V}_\mathfrak{f}}[1,\mathbf{n},\mathbb{I}_{-1,1}[\mathbf{n}],\mathbf{0}]\,\omega=\\&\quad=\left(\mathsf{tm}\left(\mathbf{W}_{0,c}[1,\mathbf{n},\mathbb{I}_{-1,1}[\mathbf{n}]]\,\omega^*\right),\ \left(\mathsf{bs}\left(\mathbf{W}_{0,c}[1,\mathbf{n},\mathbb{I}_{-1,1}[\mathbf{n}]]\,\omega^*\right),c\right)\right)=\\&\quad=\left(\mathsf{tm}(\omega^*),\ \left(\mathsf{bs}(\omega^*),c\right)\right)=(t,\ (x,c))=\omega,\end{aligned}$$

that was necessary to prove. From the equality (46) it follows, that $\mathbb{I}_{\mathcal{M}\left(\mathfrak{H}_{\mathfrak{V}_\mathfrak{f}}\right)}\in\mathfrak{PT}_+(\mathfrak{H};\mathfrak{V}_\mathfrak{f})$. Besides this, in accordance with Remark 7, $\mathbb{I}_{\mathcal{M}\left(\mathfrak{H}_{\mathfrak{V}_\mathfrak{f}}\right)}[\mathcal{B}]=\mathcal{B}$. Hence, by Property 7(1), we can define the reference frame:

$$\mathfrak{l}_0=\left(\mathbb{I}_{\mathcal{M}\left(\mathfrak{H}_{\mathfrak{V}_\mathfrak{f}}\right)},\mathbb{I}_{\mathcal{M}\left(\mathfrak{H}_{\mathfrak{V}_\mathfrak{f}}\right)}[\mathcal{B}]\right)=\left(\mathbb{I}_{\mathcal{M}\left(\mathfrak{H}_{\mathfrak{V}_\mathfrak{f}}\right)},\mathcal{B}\right)\in\mathcal{L}k\left(\mathfrak{KPT}(\mathfrak{H},\mathcal{B};\mathfrak{V}_\mathfrak{f})\right).$$

Now, we fix any reference frame $\mathfrak{l}=(U,U[\mathcal{B}])\in\mathcal{L}k\left(\mathfrak{KPT}(\mathfrak{H},\mathcal{B};\mathfrak{V}_\mathfrak{f})\right)$, where $U=\mathbf{W}_{\lambda,\mathfrak{V}_\mathfrak{f}}[1,\mathbf{n},J;\mathbf{a}]\in\mathfrak{PT}_+(\mathfrak{H};\mathfrak{V}_\mathfrak{f})$.

According to Properties 7(3, 5), we obtain:

$$\mathbb{M}k(\mathfrak{l}) = \mathbb{R}\times \mathbf{Zk}\left(\widehat{\mathfrak{H}}\right) = \mathbb{R}\times\mathfrak{H} = \mathcal{M}\left(\mathfrak{H}\right); \tag{47}$$

$$\langle !\,\mathfrak{l}\leftarrow\mathfrak{l}_0\rangle\,\omega = U\left(\mathbb{I}^{[-1]}_{\mathcal{M}\left(\mathfrak{H}_{\mathfrak{V}_{\mathfrak{f}}}\right)}\omega\right) = U\omega = \mathbf{W}_{\lambda,\mathfrak{V}_{\mathfrak{f}}}\left[1,\mathbf{n},J;\mathbf{a}\right]\omega \tag{48}$$

$$\left(\forall\omega\in\mathbb{B}\mathfrak{s}\left(\mathfrak{l}_0\right) = \mathbb{B}\mathfrak{s}(\mathcal{B})\subseteq\mathcal{M}\left(\mathfrak{H}_{\mathfrak{V}_{\mathfrak{f}}}\right)\right.$$

Using Property 7(3) as well as equality (18), for an elementary-time state $\omega = (t,(x,c))\in\mathbb{B}\mathfrak{s}(\mathfrak{l})$ we get:

$$\mathbf{Q}^{\langle\mathfrak{l}\rangle}(\omega) = \left(\mathsf{tm}\left(\omega\right),\mathbf{q}(\mathsf{bs}\left(\omega\right))\right) = \left(t,\mathbf{q}\left((x,c)\right)\right) = (t,x) = \omega^*. \tag{49}$$

Hence, using Definition 5 (item 1) and equality (17), we deduce:

$$\mathbf{Q}^{\langle\mathfrak{l}\leftarrow\mathfrak{l}_0\rangle}(\omega) = \mathbf{Q}^{\langle\mathfrak{l}\rangle}\left(\langle !\,\mathfrak{l}\leftarrow\mathfrak{l}_0\rangle\,\omega\right) = \left(\mathbf{W}_{\lambda,\mathfrak{V}_{\mathfrak{f}}}\left[1,\mathbf{n},J;\mathbf{a}\right]\omega\right)^* =$$
$$= \mathbf{W}_{\lambda,c}\left[1,\mathbf{n},J;\mathbf{a}\right]\omega^*\quad\left(\forall\omega\in\mathbb{B}\mathfrak{s}\left(\mathfrak{l}_0\right) = \mathbb{B}\mathfrak{s}(\mathcal{B})\subseteq\mathcal{M}\left(\mathfrak{H}_{\mathfrak{V}_{\mathfrak{f}}}\right)\right). \tag{50}$$

2. By conditions of Theorem a number $\eta>0$ exists such, that $\mathfrak{V}_{\mathfrak{f}}\subseteq[\eta,\infty)$.

2.1. Suppose, that there exist elementary states $\widetilde{x}_1 = (x_1,c_1)$, $\widetilde{x}_2 = (x_2,c_2)\in\mathfrak{B}\mathfrak{s}(\mathcal{B})$ such, that $c_1\neq c_2$. Since, by Property 2(2), $\mathfrak{B}\mathfrak{s}(\mathcal{B}) = \left\{\mathsf{bs}\left(\omega\right)\mid\omega\in\mathbb{B}\mathfrak{s}(\mathcal{B})\right\}$, then there exist elementary-time states of kind $\omega_1 = (t_1,\widetilde{x}_1) = (t_1,(x_1,c_1))\in\mathbb{B}\mathfrak{s}(\mathcal{B})$, $\omega_2 = (t_2,\widetilde{x}_2) = (t_2,(x_2,c_2))\in\mathbb{B}\mathfrak{s}(\mathcal{B})$. Now, we consider two cases.

Case 2.1.1: $\omega_1^*\neq\omega_2^*$. Consider any fixed operator $J_1\in\mathfrak{U}\left(\mathfrak{H}_1\right)$. By Lemma 1, there exist $\lambda_1\in(0,\eta)$, $\mathbf{n}_1\in\mathbf{B}_1\left(\mathfrak{H}_1\right)$ and $\mathbf{a}_1\in\mathcal{M}\left(\mathfrak{H}\right)$, such, that

$$\mathbf{W}_{\lambda_1,c_1}\left[1,\mathbf{n}_1,J_1;\mathbf{a}_1\right]\omega_1^* = \mathbf{W}_{\lambda_1,c_2}\left[1,\mathbf{n}_1,J_1;\mathbf{a}_1\right]\omega_2^*. \tag{51}$$

Let us introduce the reference frame:

$$\mathfrak{l}_1 := \left(U_1,U_1[\mathcal{B}]\right)\in\mathcal{L}k\left(\mathfrak{K}\mathfrak{P}\mathfrak{T}\left(\mathfrak{H},\mathcal{B};\mathfrak{V}_{\mathfrak{f}}\right)\right),\quad\text{where}$$
$$U_1 := \mathbf{W}_{\lambda_1;\mathfrak{V}_{\mathfrak{f}}}\left[1,\mathbf{n}_1,J_1,\mathbf{a}_1\right]\in\mathfrak{P}\mathfrak{T}_+\left(\mathfrak{H};\mathfrak{V}_{\mathfrak{f}}\right).$$

According to (50), and (51), we receive:

$$\mathbf{Q}^{\langle\mathfrak{l}_1\leftarrow\mathfrak{l}_0\rangle}\left(\omega_1\right) = \mathbf{W}_{\lambda_1,c_1}\left[1,\mathbf{n}_1,J_1;\mathbf{a}_1\right]\omega_1^* = \mathbf{W}_{\lambda_1,c_2}\left[1,\mathbf{n}_1,J_1;\mathbf{a}_1\right]\omega_2^* = \mathbf{Q}^{\langle\mathfrak{l}_1\leftarrow\mathfrak{l}_0\rangle}\left(\omega_2\right).$$

From the other hand, by the formula (49), we obtain $\mathbf{Q}^{\langle\mathfrak{l}_0\rangle}\left(\omega_1\right) = \omega_1^*\neq\omega_2^* = \mathbf{Q}^{\langle\mathfrak{l}_0\rangle}\left(\omega_2\right)$. Thus, for the elementary-time states ω_1,ω_2 we have $\mathbf{Q}^{\langle\mathfrak{l}\leftarrow\mathfrak{l}_0\rangle}\left(\omega_1\right) = \mathbf{Q}^{\langle\mathfrak{l}\leftarrow\mathfrak{l}_0\rangle}\left(\omega_2\right)$, while $\mathbf{Q}^{\langle\mathfrak{l}_0\rangle}\left(\omega_1\right)\neq\mathbf{Q}^{\langle\mathfrak{l}_0\rangle}\left(\omega_2\right)$. Hence, by Theorem 1, the reference frames $\mathfrak{l}_0$ and $\mathfrak{l}$ do not allow universal coordinate transform. Therefore, in accordance with Assertion 6, item 2, the kinematic set $\mathfrak{K}\mathfrak{P}\mathfrak{T}\left(\mathfrak{H},\mathcal{B};\mathfrak{V}_{\mathfrak{f}}\right)$ do not allow universal coordinate transform in this case.

Case 2.1.2: $\omega_1^* = \omega_2^*$. Consider any fixed operator $J_2\in\mathfrak{U}\left(\mathfrak{H}_1\right)$ and vector $\mathbf{n}_2\in\mathbf{B}_1\left(\mathfrak{H}_1\right)$. According to Corollary 4, there exist $\lambda_2\in(0,\eta)$ and $\mathbf{a}_2\in\mathcal{M}\left(\mathfrak{H}\right)$, such, that

$$\mathbf{W}_{\lambda_2,c_1}\left[1,\mathbf{n}_2,J_2;\mathbf{a}_2\right]\omega_1^*\neq\mathbf{W}_{\lambda_2,c_2}\left[1,\mathbf{n}_2,J_2;\mathbf{a}_2\right]\omega_2^*. \tag{52}$$

Let us consider the reference frame:

$$\mathfrak{l}_2 : = \left(U_2,U_2[\mathcal{B}]\right)\in\mathcal{L}k\left(\mathfrak{K}\mathfrak{P}\mathfrak{T}\left(\mathfrak{H},\mathcal{B};\mathfrak{V}_{\mathfrak{f}}\right)\right),\quad\text{where}$$
$$U_2 = \mathbf{W}_{\lambda_2;\mathfrak{V}_{\mathfrak{f}}}\left[1,\mathbf{n}_2,J_2,\mathbf{a}_2\right]\in\mathfrak{P}\mathfrak{T}_+\left(\mathfrak{H};\mathfrak{V}_{\mathfrak{f}}\right).$$

According to (50), and (52), we receive:

$$\mathbf{Q}^{\langle\mathfrak{l}_2\leftarrow\mathfrak{l}_0\rangle}\left(\omega_1\right) = \mathbf{W}_{\lambda_2,c_1}\left[1,\mathbf{n}_2,J_2;\mathbf{a}_2\right]\omega_1^*\neq\mathbf{W}_{\lambda_2,c_2}\left[1,\mathbf{n}_2,J_2;\mathbf{a}_2\right]\omega_2^* = \mathbf{Q}^{\langle\mathfrak{l}_2\leftarrow\mathfrak{l}_0\rangle}\left(\omega_2\right).$$

From the other hand, by the formula (49), we obtain: $\mathbf{Q}^{\langle\mathfrak{l}_0\rangle}\left(\omega_1\right) = \omega_1^* = \omega_2^* = \mathbf{Q}^{\langle\mathfrak{l}_0\rangle}\left(\omega_2\right)$.

Thus, for the elementary-time states ω_1,ω_2 we have $\mathbf{Q}^{\langle \mathfrak{l}_2 \leftarrow \mathfrak{l}_0 \rangle}(\omega_1) \neq \mathbf{Q}^{\langle \mathfrak{l}_2 \leftarrow \mathfrak{l}_0 \rangle}(\omega_2)$, while $\mathbf{Q}^{\langle \mathfrak{l}_0 \rangle}(\omega_1) = \mathbf{Q}^{\langle \mathfrak{l}_0 \rangle}(\omega_2)$. Hence, by Theorem 1, the reference frames $\mathfrak{l}_0$ and $\mathfrak{l}_2$ do not allow universal coordinate transform. Therefore, in accordance with Assertion 6, item 2, the kinematic set $\mathfrak{KPT}(\mathfrak{H}, \mathcal{B}; \mathfrak{V}_\mathfrak{f})$ does not allow universal coordinate transform.

Thus, if the kinematic set $\mathfrak{KPT}(\mathfrak{H}, \mathcal{B}; \mathfrak{V}_\mathfrak{f})$ allows universal coordinate transform, then there not exist elementary states $\widetilde{x}_1 = (x_1, c_1)$, $\widetilde{x}_2 = (x_2, c_2) \in \mathfrak{Bs}(\mathcal{B})$ such, that $c_1 \neq c_2$.

2.2. Now we suppose, that in base changeable set $\mathcal{B}$ there not exist elementary states $\widetilde{x}_1 = (x_1, c_1)$, $\widetilde{x}_2 = (x_2, c_2) \in \mathfrak{Bs}(\mathcal{B})$ such, that $c_1 \neq c_2$. Under this assumption a number $c_0 \in \mathfrak{V}_\mathfrak{f}$ must exist such, that arbitrary elementary state $\widetilde{x} \in \mathfrak{Bs}(\mathcal{B})$ can be represented in the form: $\widetilde{x} = (x, c_0)$, where $x \in \mathfrak{H}$. Chose any reference frame:

$$\begin{aligned} &\mathfrak{l} := (U, U[\mathcal{B}]) \in \mathcal{L}k\left(\mathfrak{KPT}(\mathfrak{H}, \mathcal{B}; \mathfrak{V}_\mathfrak{f})\right), \quad \text{where} \\ &U = \mathbf{W}_{\lambda;\mathfrak{V}_\mathfrak{f}}[1, \mathbf{n}, J, \mathbf{a}] \in \mathfrak{PT}_+(\mathfrak{H}; \mathfrak{V}_\mathfrak{f}). \end{aligned}$$

According to (50), (49), for arbitrary elementary-time state $\omega = (t, (x, c_0)) \in \mathbb{B}\mathfrak{s}(\mathfrak{l}_0) = \mathbb{B}\mathfrak{s}(\mathcal{B})$ we obtain:

$$\mathbf{Q}^{\langle \mathfrak{l} \leftarrow \mathfrak{l}_0 \rangle}(\omega) = \mathbf{W}_{\lambda,c_0}[1, \mathbf{n}, J; \mathbf{a}]\,\omega^* = \mathbf{W}_{\lambda,c_0}[1, \mathbf{n}, J; \mathbf{a}]\left(\mathbf{Q}^{\langle \mathfrak{l}_0 \rangle}(\omega)\right),$$

where $\mathbf{W}_{\lambda,c_0}[1, \mathbf{n}, J; \mathbf{a}]$ is a bijection from $\mathcal{M}(\mathfrak{H})$ onto $\mathcal{M}(\mathfrak{H})$ (and, by (47), $\mathbf{W}_{\lambda,c_0}[1, \mathbf{n}, J; \mathbf{a}]$ is a bijection from $\mathbb{M}k(\mathfrak{l}_0)$ onto $\mathbb{M}k(\mathfrak{l})$). Hence, in accordance with Definition 5, the mapping $\mathbf{W}_{\lambda,c_0}[1, \mathbf{n}, J; \mathbf{a}]$ is universal coordinate transform from $\mathfrak{l}_0$ to $\mathfrak{l}$. Consequently, the reference frames $\mathfrak{l}_0$ and $\mathfrak{l}$ allow universal coordinate transform, ie $\mathfrak{l}_0 \rightleftarrows \mathfrak{l}$ (for any reference frame $\mathfrak{l} \in \mathcal{L}k\left(\mathfrak{KPT}(\mathfrak{H}, \mathcal{B}; \mathfrak{V}_\mathfrak{f})\right)$). Thus, by Assertion 6, kinematic set $\mathfrak{KPT}(\mathfrak{H}, \mathcal{B}; \mathfrak{V}_\mathfrak{f})$ allows universal coordinate transform. $\square$

Similarly to Theorem 6 it can be proved the following theorem.

Theorem 7. *Let the set of forbidden velocities $\mathfrak{V}_\mathfrak{f} \subseteq (0, \infty]$ be separated from zero (ie there exists a number $\eta > 0$ such, that $\mathfrak{V}_\mathfrak{f} \subseteq [\eta, \infty]$).*

Kinematic set $\mathfrak{KPT}_0(\mathfrak{H}, \mathcal{B}; \mathfrak{V}_\mathfrak{f})$ allows universal coordinate transform if and only if there don't exist elementary states $\widetilde{x}_1 = (x_1, c_1)$, $\widetilde{x}_2 = (x_2, c_2) \in \mathfrak{Bs}(\mathcal{B})$ such, that $c_1 \neq c_2$.

Note, that theorems 6 and 7 were announced in the paper [31].

8 Conclusions.

Development of kinematic theories of tachyon movement (which is especially intensified in the recent years) generates the problem of building a new mathematical apparatus, which would allow to investigate evolution of physical systems in a framework of different laws of kinematics. Concerning to the given problem in this paper the following results are obtained:

1. The definitions of actual and universal coordinate transform in kinematic sets are presented.

2. The kinematic sets of kind $\mathfrak{KP}(\mathfrak{H}, \mathcal{B}, c)$, $\mathfrak{KP}_0(\mathfrak{H}, \mathcal{B}, c)$, $\mathfrak{KPT}(\mathfrak{H}, \mathcal{B}, c)$, $\mathfrak{KPT}_0(\mathfrak{H}, \mathcal{B}, c)$ are constructed. These kinematic sets represent mathematically strict models of evolution of physical systems in the framework of kinematics of special relativity theory as well as it's tachyon extension based on the generalized Lorentz-Poincare transformations (in the sense of E. Recami).

3. The kinematic sets of kind $\mathfrak{KPT}(\mathfrak{H}, \mathcal{B}; \mathfrak{V}_\mathfrak{f})$ and $\mathfrak{KPT}_0(\mathfrak{H}, \mathcal{B}; \mathfrak{V}_\mathfrak{f})$ are constructed. These kinematic sets may simulate evolution of physical systems under the condition of hypothesis on existence of particle-dependent velocity of light.

4. It is proved, that the kinematic sets of type $\mathfrak{KP}(\mathfrak{H}, \mathcal{B}, c)$, $\mathfrak{KP}_0(\mathfrak{H}, \mathcal{B}, c)$, $\mathfrak{KPT}(\mathfrak{H}, \mathcal{B}, c)$, $\mathfrak{KPT}_0(\mathfrak{H}, \mathcal{B}, c)$ allow universal coordinate, whereas in the kinematic sets $\mathfrak{KPT}(\mathfrak{H}, \mathcal{B}; \mathfrak{V}_\mathfrak{f})$, $\mathfrak{KPT}_0(\mathfrak{H}, \mathcal{B}; \mathfrak{V}_\mathfrak{f})$ universal coordinate transform can not exist in non-trivial cases.

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